ELECTRO-HYDRODYNAMIC (EHD) THRUSTER ANALYSIS AND OPTIMIZATION

by

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This thesis was prepared under the direction of the Candidate’s Thesis Advisor and has received approval. It was submitted to the Dean of the School of Engineering and the full Faculty, and was approved as partial fulfillment of the requirements of the degree of Master of Engineering.

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In every sense of the words; I love my parents. I deeply appreciate their obvious – and not-so obvious – contributions to my education. Thank you.
Abstract

This paper presents the thrust optimization of Electro-hyrdodynamic (EHD) thrusters through theoretical, experimental, and simulation approaches. The underlying force production theory is explained with electrical concepts, such as corona discharge, electrode polarity, Corona Inception Voltage (CIV), and air breakdown principles. These concepts are applied to the development of the theoretical derivation of the thrust equation relating the design variables in an EHD Thruster. The theoretical relationships derived were used to approximate an EHD Thruster with the optimal force-to-weight ratio.

Using the guiding design principles, a successful proof-of-concept EHD lifter model was built and tested with a digital power supply under the appropriate laboratory safety conditions. Although many of the design hypotheses were theoretically-sound, an updated set of conclusions were expressed to explain and improve the failed trials with different thruster geometries, electrode sizes, collector shapes, and various materials.

To avoid further safety hazards involving high voltage and corona discharge, a simulation was created using COMSOL Multiphysics 3.4. The electrostatics, charge transport, and hydrodynamics governing equations were implemented with the software to obtain graphical representations of the electric field and velocity airflow distributions of 2-D and 3-D thruster models. Although these simulation force calculations are consistent with the theoretical and experimental results, the limitations in hardware resources prevented a full optimization analysis with complex three-dimensional models. Further integration with COMSOL scripting tools has a promising outlook for future improvements.
Contents

Chapter 1: Statement of Purpose .............................................................................................................................. 1
  1.1 Introduction .................................................................................................................................................... 1
  1.2 Proposed Research ........................................................................................................................................ 4
  1.3 Research Objectives ..................................................................................................................................... 6
Chapter 2: Fundamental EHD Theory ..................................................................................................................... 8
  2.1 Basic EHD Thruster Principle ....................................................................................................................... 8
  2.2 Detailed Steps of Theory .............................................................................................................................. 10
  2.3 Corona Discharge ........................................................................................................................................ 20
  2.4 Electrode Polarity ........................................................................................................................................ 22
  2.5 Air Breakdown ............................................................................................................................................ 24
  2.6 Corona Inception Voltage (CIV) .................................................................................................................. 25
  2.7 Ionization Layer .......................................................................................................................................... 26
Chapter 3: EHD Thruster Derivation ........................................................................................................................ 28
  3.1 Ion momentum transfer model ..................................................................................................................... 28
  3.2 EHD Flow model ........................................................................................................................................ 30
  3.3 Barsoukov’s model ...................................................................................................................................... 32
Chapter 4: EHD Thruster Design ........................................................................................................................... 35
  4.1 Following Barsoukov’s Model ....................................................................................................................... 36
  4.2 EHD Thruster Geometry ............................................................................................................................. 37
  4.3 Electrode-Collector Air Gap ......................................................................................................................... 39
  4.4 Collector Curvature ..................................................................................................................................... 40
  4.5 Power Consumption ................................................................................................................................... 42
  4.6 Thruster Design Summary ........................................................................................................................... 43
Chapter 5: EHD Thruster Experiments .................................................................................................................... 45
  5.1 Safety Considerations ................................................................................................................................. 46
  5.2 Experimental Setup .................................................................................................................................... 48
  5.3 Experimental Results ................................................................................................................................. 52
  5.4 Experimental Maximize Thrust Conclusions ............................................................................................... 53
Chapter 6: EHD Governing Equations ................................................................. 56
  6.1 Electrostatic Equations ................................................................................. 56
  6.2 Fluid Dynamic Equations ........................................................................... 61
Chapter 7: Thruster Modeling with COMSOL Multiphysics 3.4 ....................... 63
  7.1 COMSOL Software ...................................................................................... 64
  7.2 EHD Modules and Setup ............................................................................. 66
  7.3 2-D Single Thruster ..................................................................................... 67
  7.4 2-D Single Thruster comparison ................................................................. 72
  7.5 3-D Single Thruster ..................................................................................... 78
  7.6 3-D Triangle Lifter ..................................................................................... 81
Chapter 8: Overall Conclusions ....................................................................... 84
  8.1 Optimization Parameters ........................................................................... 84
  8.2 Future Work with Simulation ................................................................. 87
  8.3 Closing Statements .................................................................................... 89
Appendices ........................................................................................................... 90
Appendix A: COMSOL Multiphysics 3.4 EHD Equations ................................. 91
  A.1 Module Equation Formulations .............................................................. 91
  A.2 Modeling Steps ......................................................................................... 93
Bibliography ......................................................................................................... 96
## List of Figures

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>Example of an asymmetric capacitor used to generate EHD thrust</td>
<td>3</td>
</tr>
<tr>
<td>2.1</td>
<td>EHD Thruster anatomy</td>
<td>9</td>
</tr>
<tr>
<td>2.2</td>
<td>Ion stream between corona electrode and collector</td>
<td>10</td>
</tr>
<tr>
<td>2.3</td>
<td>Steady state EHD Thruster without applied voltage</td>
<td>10</td>
</tr>
<tr>
<td>2.4</td>
<td>Apply ~20kV to EHD Thruster system and focus on Impact Ionization</td>
<td>11</td>
</tr>
<tr>
<td>2.5</td>
<td>Steady state ionization and drift layers</td>
<td>13</td>
</tr>
<tr>
<td>2.6</td>
<td>Drift layer and elastic collisions</td>
<td>14</td>
</tr>
<tr>
<td>2.7</td>
<td>Momentum transfer and space charge</td>
<td>15</td>
</tr>
<tr>
<td>2.8</td>
<td>Space charge effect</td>
<td>16</td>
</tr>
<tr>
<td>2.9</td>
<td>Force body diagram of Ion-wind</td>
<td>18</td>
</tr>
<tr>
<td>2.10</td>
<td>Corona discharge and Electrical arcing</td>
<td>22</td>
</tr>
<tr>
<td>2.11</td>
<td>Representation of actual and effective electrode radius</td>
<td>23</td>
</tr>
<tr>
<td>4.1</td>
<td>Non-uniform distribution of electric field</td>
<td>36</td>
</tr>
<tr>
<td>4.2</td>
<td>Force vs air gap for increasing breakdown voltages</td>
<td>39</td>
</tr>
<tr>
<td></td>
<td>(Left) Force vs air gap for a fixed voltage</td>
<td>39</td>
</tr>
<tr>
<td>5.1</td>
<td>Photos of experimental setup with safety</td>
<td>47</td>
</tr>
<tr>
<td>5.2</td>
<td>Experiments leading to proof-of-concept flight</td>
<td>49</td>
</tr>
<tr>
<td>5.3</td>
<td>Single thruster experiment</td>
<td>51</td>
</tr>
<tr>
<td>5.3</td>
<td>Force vs Voltage focus on quadratic relationship</td>
<td>52</td>
</tr>
<tr>
<td>7.1</td>
<td>Steps for COMSOL</td>
<td>66</td>
</tr>
<tr>
<td>7.2</td>
<td>2-D single thruster model</td>
<td>67</td>
</tr>
</tbody>
</table>
Figure 7.3. Electric potential for 2-D model ................................................................. 68
Figure 7.4. Space charge density for 2-D model .......................................................... 69
Figure 7.5. Electric field for 2-D model ....................................................................... 69
Figure 7.6. Velocity distribution for 2-D model .......................................................... 70
Figure 7.7. Coulomb force ......................................................................................... 70
Figure 7.8. Pressure distribution for 2-D model .......................................................... 71
Figure 7.9. Single Thruster for comparison ................................................................. 72
Figure 7.10. Force vs Voltage comparison ................................................................. 73
Figure 7.11. Comparison electric field and space charge density .................................. 74
Figure 7.12. Comparison for velocity distribution ...................................................... 75
Figure 7.13. Force vs Voltage for air gaps in 3D graph .............................................. 76
Figure 7.14. Force vs Voltage for air gaps ................................................................. 76
Figure 7.15. Force vs Air gap .................................................................................... 77
Figure 7.16. 3-D Single thruster ............................................................................... 78
Figure 7.17. 3-D Single thruster electric potential ..................................................... 79
Figure 7.18. 3-D Single thruster space charge density .............................................. 80
Figure 7.19. 3-D Single thruster electric field ............................................................. 80
Figure 7.20. 3-D Triangle Thruster ............................................................................. 82
Figure 7.21. 3-D Triangle Thruster Electric Potential ............................................... 82
Table of Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description with units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$CIV$</td>
<td>Corona Inception Voltage (V)</td>
</tr>
<tr>
<td>$d$</td>
<td>Distance between the positive and negative electrodes, or air gap (m)</td>
</tr>
<tr>
<td>$D$</td>
<td>Charge diffusion coefficient of ions (5.3x10^{-5} m^2/s)</td>
</tr>
<tr>
<td>$E_0$</td>
<td>Electric field strength necessary to break down air (V/m)</td>
</tr>
<tr>
<td>$E_i$</td>
<td>Electric field strength necessary for corona discharge (V/m)</td>
</tr>
<tr>
<td>$\vec{E}$</td>
<td>Electric field intensity vector (V/m)</td>
</tr>
<tr>
<td>$F$</td>
<td>Force (N)</td>
</tr>
<tr>
<td>$f_s$</td>
<td>Other body forces from Navier-Stokes equation (N)</td>
</tr>
<tr>
<td>$g$</td>
<td>Acceleration due to gravity (= 9.81 m/s^2)</td>
</tr>
<tr>
<td>$I$</td>
<td>Current (Amperes)</td>
</tr>
<tr>
<td>$\vec{j}$</td>
<td>Ionic Current Density Vector</td>
</tr>
<tr>
<td>$L$</td>
<td>Length of collector (m)</td>
</tr>
<tr>
<td>$m$</td>
<td>Mass of particle (kg)</td>
</tr>
<tr>
<td>$p$</td>
<td>Static air pressure (Pascals)</td>
</tr>
<tr>
<td>$q$</td>
<td>Charge of particle (C)</td>
</tr>
<tr>
<td>$r_0$</td>
<td>Effective wire radius (m)</td>
</tr>
<tr>
<td>$r_c$</td>
<td>Radius of curvature of collector (m)</td>
</tr>
<tr>
<td>$r_w$</td>
<td>Actual electrode radius (m)</td>
</tr>
<tr>
<td>$\vec{U}$</td>
<td>Velocity vector of air flow</td>
</tr>
<tr>
<td>$V$</td>
<td>Applied voltage between electrodes (V)</td>
</tr>
<tr>
<td>$v_d$</td>
<td>Drift velocity (m/s)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Peek’s value (0.301√cm or 0.0301√m)</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Air density factor</td>
</tr>
<tr>
<td>$\varepsilon_0$</td>
<td>Permittivity of free space (= 8.854x10^{-12} C/Vm)</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Air dynamic viscosity ($= 1.8205 \times 10^{-5}$ Ns/m²)</td>
</tr>
<tr>
<td>$\mu_i$</td>
<td>General ion mobility (cm²/Vs)</td>
</tr>
<tr>
<td>$\mu_{iN}$</td>
<td>Ion mobility of nitrogen ($= 2.5 \ \text{cm}^2/\text{Vs}$)</td>
</tr>
<tr>
<td>$\mu_n$</td>
<td>Ion mobility of a negative ion ($= 2.7 \times 10^{-4}$ m²/Vs)</td>
</tr>
<tr>
<td>$\mu_p$</td>
<td>Ion mobility of a positive ion ($= 2.0 \times 10^{-4}$ m²/Vs)</td>
</tr>
<tr>
<td>$\pi$</td>
<td>Pi ($= 3.14159265\ldots$)</td>
</tr>
<tr>
<td>$\rho_{air}$</td>
<td>Air density ($= 1.23$ kg/m³)</td>
</tr>
<tr>
<td>$\rho_{ion}$</td>
<td>Space charge density (C/m³)</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>Scalar electric potential (V)</td>
</tr>
</tbody>
</table>
Chapter 1:

Statement of Purpose

1.1 Introduction

The study of electro-hydrodynamics (EHD) is a particular domain of electrodynamics concerned with fluid flow in the presence of electric forces in dielectric media. The dielectric fluid (in this case, dry air) exhibits very low conductivity, thus having the ability to sustain high electric fields with small currents. The high electric fields in these fluids adhere to the fundamental properties of ionization and space charge distribution required to assist pressure manipulation and fluid flow. EHD concepts are relatively mature and have been researched since the 18\textsuperscript{th} century. The first instance of an “ion-wind” created by the ionization and acceleration of air molecules between the electrodes of a high-voltage source was observed by Francis Hauksbee [1]. Since then, researchers have explored the use of “ion-wind” for air-filtration devices, solid-fluid boundary layer modification [2], cooling of integrated circuits [3,4], electro-hydrodynamic (EHD) pumping [5-6], particulate removal in Electrostatic Precipitators (ESP) [7-11], EHD Jet printing, electro-acoustics (EHD speakers) [12], and propulsion applications [13]. Recent research in this field has produced a growing interest in utilizing the thrust resulting from a high dc voltage across an asymmetric capacitor as a means of propulsion.
A major milestone in EHD propulsion occurred in 1964, when Alexander de Seversky built and demonstrated a device called the Ionocraft, which could hover above the ground, without moving parts, utilizing ion-wind propulsion. His invention was patented, and a *Popular Mechanics* article described the device’s theory of operation and possible future applications [14]. The article mentions projected uses for the technology, including military reconnaissance and rescue, airborne traffic monitors, and uses in commuter transport. Many of the advantageous claims for thruster scalability, speed, and power efficiency discussed in the article were appealing, but incomplete in analysis. Unfortunately, interest in EHD propulsion technology waned in the following years due to difficulties in increasing thrust, building a lightweight high-voltage power supply, designing a control system for the device, and maintaining safety regulations for commercial usage.

However, EHD propulsion technology remains popular among university students in physics and electrical engineering fields and independent researchers who make their experimental results and thruster construction guidelines available to the public to encourage further experimentation with EHD Thruster technology [15]. The conclusions from these experiments were used as a foundation for our initial designs and proof-of-concept models.

The greatest advantage of an EHD propulsion system over other technologies is its lack of moving parts, which makes it easy to manufacture and increases its reliability. EHD Thrusters only use electrical energy, and allow nearly silent hovering. Although high voltage is required, the low current yields high power efficiency in many of the experiments conducted for EHD micro-pump and ESP applications [5, 10]. The construction materials for the anatomy of EHD Thruster designs are inexpensive, but must be created with high precision. Research on maximizing thrust and testing the sensitivity of adjustments in the
physical anatomy of the asymmetrical capacitor have been ongoing and rigorous in order to optimize the technology that operates without fuel, a propellant, or combustion [16].

EHD Lifters are proof-of-concept models that levitate due to the propulsion generated by EHD Thrusters. Based on initial research and our reported experiments in Chapter 5, the force generated by these lifters can cause levitation, but this force is not substantial enough to maintain payloads greater than 200g. The scalability of the thrust-to-weight ratio has not been realistically tested because the variables that affect the thrust are very sensitive and can drastically change when involving complex electric field interactions utilizing thrust improvement configurations, such as nesting or stacking.

![Figure 1.1. Example of an asymmetric capacitor used to generate EHD thrust](image)

EHD Thruster technology refers to the propulsion produced by a high voltage applied to an asymmetric capacitor configuration seen in Fig. 1.1. The presence of the high voltage across the thrust-production device places restrictions on the applications due to the interference of strong electric fields with other electronic and conductive devices. The safety hazards involving high voltage and ozone generation (discussed in Section 5.1) are a significant risk to the operator and possible passengers – not to mention, its lack of a “green” initiative to save the environment. In addition, designing a scalable and controllable EHD
Thruster engine system faces its main difficulty when focusing on the high voltage power source. To power the EHD propulsion system, either batteries or a portable generator would be required. The former would need a massive power supply to support the high voltage operational requirements, and the latter would be inefficient since most generators require an independent fuel source. The power supply used to supply the high voltage must also accurately control the voltage and current values that support the device’s control system.

The existence of these disadvantages does not mean that the underlying technology should be disregarded. Looking back ten years, could we have imagined the phenomenal advances in every aspect of science today? Were there not hundreds-of-thousands of ideas regarded as impossible half of a century ago, but now proven practical within the past decade? Ground-breaking innovations specific to improving insulation for electric fields and regulating high voltages can make commercial usage of EHD Thrusters viable.

A reliance on this type of break-through is wishful thinking, but an advanced analysis of the underlying physical properties of this phenomenon combined with a computer simulation of these complex configurations can begin to answer many of the questions regarding scalability, precision, affects of environment, and boundaries on operation conditions.

### 1.2 Proposed Research

To maximize the thrust produced by the EHD Lifters, the electrical and physical variables of the EHD Thrusters were altered and tested to extrapolate optimal points. Unfortunately, building and testing the EHD lifter models is time consuming and requires high precision. Testing all of these variables independently yields dangerous exposure to the
high voltage, ozone generation, and UV radiation produced by the corona discharge phenomenon that causes the thrust production. Physical testing would eventually be required to test variables in the environment that were not accounted for by the physics equations, or to confirm the simulation’s underlying governing equations, but the more realistic measure of optimization should proceed with the design and simulation phases.

The presence of corona phenomena further complicates the mathematical models used to simulate the impact of the electric field and current density distributions in inter-electrode space. The space charge distribution may be computed analytically only for particular symmetry arrangements under some simplifying assumptions. Several models have been proposed for calculating the electric field and charge density distribution using the finite difference method, finite element techniques, boundary element method with method of characteristics, and combined boundary element with finite difference method [7,17-18]. Current simulations from electrostatic precipitators, particulate removal in EHD air filtration, and ion-drag pumps are represented by governing equations from Poisson’s equation, charge density distribution, Navier-Stokes fluid dynamics equations, and Peek’s equation [8,9,19-21].

Applications of these simulations to EHD Thruster theory have confirmed the fluid flow operation, but have been inconclusive regarding the optimization of variables and materials to maximize thrust. In addition, the issue of scalability for complex thruster improvement techniques poses a unique challenge. Complex electric field interactions between the corona and collector electrodes lead to design difficulties and trial-error testing, instead of thorough analysis.
1.3 Research Objectives

This study is comprised of four workflow phases. The first phase (Chapter 2) involves a thorough research of the fundamental physics surrounding the corona discharge phenomenon that causes the EHD thrust. Electrical concepts, including Corona Inception Voltage (CIV) (Section 2.6), corona discharge (Section 2.3), ionization layer (Section 2.5), electrical arcing (Section 2.3), and electrode polarity (Section 2.4) were studied to produce boundary conditions for the phenomenon. Existing theoretical derivations (Chapter 3) modeling the thrust production to the EHD Thruster design variables were used to suggest optimal design parameters for experimental testing. These design parameters (Chapter 4) include thruster geometry, air gap, collector radius of curvature, and power consumption.

The second phase (Chapter 5) involves the construction of EHD Lifters to achieve a proof-of-concept flight. Variables, such as the air gap, applied voltage, thruster geometry, and corona wire radius, were adjusted to maximize thrust. Conclusions from these experiments were used for comparison in the subsequent simulation phase.

The third phase involves the research of the governing equations (Chapter 6) of the corona discharge phenomenon to build a complex simulator utilizing the COMSOL Multiphysics 3.4 software. This simulation offers the ability to calculate the complex relationships within any thruster geometry to determine thrust production. The COMSOL Multiphysics analysis is unique to the EHD Thruster model in order to confirm and extend measurements of the performance theories proposed by existing and prior experiments. A detailed study of the airflow and electric field forces affecting the specific geometries allows for thrust optimization. Computer models of new thruster designs could be imported into the
software interface to test different sub-domain settings and boundary conditions for the generated thrust from the corona discharge phenomenon.

The final phase (Chapter 7) of the experiment is to compare, and account for, the differences between the experimental and simulation results. The initial optimization of the force-per-unit-length produced by the phenomenon is a 2-D EHD Thruster in order to reduce the computation time and confirm thrust generation approximations from the earlier theoretical derivations. Electrical and fluid mechanics distribution are analyzed to determine the subsequent effects of altering specific thruster variables.

The optimized 2-D model is then extruded to produce a 3-D EHD Thruster model to compare the complex electric field interactions around corners and in stacked formations. The resulting optimized 3-D thruster variables would be used to emulate EHD Lifter models. In particular, the triangle design was tested for theoretical thrust production.
Chapter 2:

Fundamental EHD Theory

EHD thrust is a particular domain of electro-hydrodynamics, which involves the study of asymmetric capacitors for the purpose of particle movement in fluid mediums. The movement of particles as a result of the EHD phenomenon can be explained through the use of basic physics, including Coulomb’s Law of Electrostatics, Conservation of Momentum, and Newton’s Third Law. The basic result is that when a high voltage (exceeding the CIV ~10kV dc) is applied across an asymmetric capacitor, the capacitor experiences a net force toward the smaller electrode [22].

This chapter will provide a detailed explanation of the EHD phenomenon for a positive corona system. Electrical properties, such as corona discharge, CIV, electrode polarity, and air breakdown will be discussed to clarify the boundary conditions restricting thrust production.

2.1 Basic EHD Thruster Principle

The aforementioned “asymmetric capacitor” refers to a formation where electric potential is applied to an electrode (called the emitter or corona electrode), which has a smaller radius of curvature than the electrode with the opposite polarity (called the collector).
The basic EHD Thruster design consists of a thin wire at positive potential suspended above a rounded metallic surface at zero or negative potential. The basic arrangement (illustrated in Fig. 2.1) labels the adjustable variables that govern thrust production.

![Figure 2.1. EHD Thruster anatomy](image)

Since electric fields are stronger at sharper points than smooth surfaces, the corona electrode will ionize the air and cause the corona discharge phenomenon at a lower voltage than the collector electrode. The ionization that occurs at the corona electrode will produce positive ions, which are then attracted to the negative collector by Coulomb's force. In the path traveling towards the collector, the positive ions collide with neutral molecules and transfer momentum to cause an ion-wind effect. By Newton's Third Law, the force downwards creates an equal and opposite force, pushing the thruster upwards similar to the concept of a helicopter. A positive corona discharge mechanism is demonstrated in Fig 2.2.
2.2 Detailed Steps of Theory

The details omitted in the previous subsection of the broader explanation of the ion-wind phenomenon will be explained thoroughly in this section. Diagrams demonstrating this process will introduce electrical concepts, which will be discussed in more detail in the following subsections.

2.2.1 Steady State without Applied Voltage

Figure 2.3. Steady state EHD Thruster without applied voltage
Before applying voltage to the EHD Thruster system, it is important to clarify the existence of positive ions and electrons occupying the inter-electrode space (Fig. 2.3). An initial concentration of electrons is required for impact ionization and subsequent corona discharge. The free electrons present in the air are a result of “background radiation” or photon excitation. This natural ionization is emitted from artificial and natural sources, such as radioactive elements and cosmic rays [23]. The majority of the ionization is caused by cosmic radiation, but radon gas released from the Earth’s crust can cause radioactive ions to attach to airborne dust and other particles. Approximately fifteen percent of this ionization is contributed from x-rays and nuclear medicine [23]. The actual percent-concentration of the electrons and ions in the free space can be disregarded for this study because the electron avalanche effect will ensure an exponential increase of electron generation, which allows for ionization to occur.

### 2.2.2 Applying Voltage to the System

![Diagram of applying voltage to an EHD Thruster system](image)

Figure 2.4. Apply ~20kV to EHD Thruster system and focus on Impact Ionization
When high voltage (exceeding the CIV) is applied to the EHD Thruster system, the corona discharge phenomenon begins to take effect. In Fig. 2.4, a positive electric potential is applied to the smaller radius corona electrode\(^1\). A high electric field is created in the gap between the positive corona electrode and the collector due to: 1) The high potential difference between these surfaces, and 2) The small radius of curvature of the corona electrode. The electric field causes free electrons in the air to accelerate towards the positive corona electrode.

A free electron can collide with a neutral air molecule with enough kinetic energy to exceed the ionization level and remove an electron from the air molecule. Thus, a free electron and a positively charged air molecule are produced as a result of this collision. This process is known as particle impact ionization [24]. The newly released electron from this process then continues to ionize more air molecules. The process where the electron-ion pairs formed from the collisions cause cascading particle impact ionizations is referred to as the electron avalanche effect.

In addition to particle impact ionization, direct electric field ionization may also occur. When the atoms or molecules come in contact with the surface of the metal electrodes, they can lose or gain a charge subject to the polarity of the electrode. As the intensity of the electric field is increased, the particles approaching the electrode are ionized before reaching it. This rate of ionization is directly dependent on the intensity of the electric field.

\(^1\) It is interesting to note that this phenomenon also occurs with the reverse electrode polarity in a similar manner (see Section 2.2.6), but this explanation will focus on a positive corona due to its high thrust production and lower energy consumption properties (see Section 2.4).
2.2.3 Steady State with layers

The steady state diagram near the corona electrode during corona discharge is shown in Fig. 2.5. At this point, the excess of free electrons (as a result of the impact ionization and electron avalanche effect) are attracted to the positive corona and form a distinction between the ionization layer and the drift layer. This layering effect at steady state is a complex and delicate balance of Coulomb attractive and repulsive forces between the positive ions, electrons, positive sheath at the boundary of the ionization layer, and the positive corona electrode. Essentially, all of the free electrons are attracted to the positive corona electrode while the positive ions feel a repulsive force from the positive corona electrode. However, complexities arise when focusing on the boundary between the ionization layer and the drift layer due to the repulsive forces of the positive sheath and free positive ions within the ionization layer. To simplify force calculations in later sections, this boundary’s properties and interaction with other layers will follow Kapstov’s Assumption (see Section 2.7).

Figure 2.5. Steady state ionization and drift layers

The ionization layer increases the effective radius of the corona electrode and maintains a new voltage on the surface of the ionization layer from the positive sheath of ions
attracted to the electrons. A mix of electrons and positive ions will occupy the ionization layer, while the drift layer ideally consists of only positive ions and neutral air molecules.

### 2.2.4 Drift Layer Dynamics

Once the air molecules are ionized, they are forced away from the corona electrode and towards the collector by Coulomb forces. In the drift layer (represented in Fig. 2.6), the positive ions collide with neutral air molecules, therefore transferring their momentum to the neutral molecules. This, in turn, creates a flow of air through the thruster (from the corona electrode to the collector). The difference in mass between the positive ion and the neutral molecule is the mass of an electron, which is practically negligible compared to the mass of the molecules. Thus, an elastic collision occurs with a full transfer of momentum.

![Image of drift layer and elastic collisions](image_url)

**Figure 2.6.** Drift layer and elastic collisions

Consider the system of positive ions as a positive ionic space charge, as shown in Fig. 2.7. As a result of this space charge collision with the “cloud” of neutral air molecules, a burst of air exits the bottom of the EHD Thruster. When the positive ions (that have not elastically collided with the neutral molecules) collide with the negatively charged collector, they are neutralized. The neutral particles that collide with the collector ideally should not
ionize (e.g., as in the case of the corona electrode) because the radius of curvature of the collector should be below the CIV.

Figure 2.7. Momentum transfer and space charge

At higher voltages, the collector electrode may start to ionize the air and cause a thrust cancellation with the formation of negative ions. More specifically, the free electrons, as a result of the impact ionization near the negative collector, will be repelled by the negative electric field. Since the electric field decreases in an inverse-squared relationship with distance, the colliding electrons lack the kinetic energy to strip neutral air molecules of their valence electrons [24]. Instead, the low energy electrons attach to the neutral air molecules to form negatively charged ions. These negative ions would cause collisions with neutral air molecules that reduce the desired magnitude of the bulk airflow.

By Newton’s Third Law, if ions force the neutral air downward, then the neutral air molecules exert an equal and opposite force upward on the ions. Due to the electric field between the electrodes, the positive ions exert a force upwards on the EHD device. In summary, the positive ions serve to transfer the energy of the electric field to neutral air molecules, which get forced out of the EHD device to create thrust.
Simultaneously while the positive ions are moving away from the positive corona, a positive ion sheath continues to form around the ionization layer. Neutral molecules that replace the empty space will be ionized by electric field ionization or particle impact ionization, and the process repeats.

### 2.2.5 The Space Charge Effect

The positive ions that drift towards the negative collector form a space charge ion-cloud, which can dynamically extend the ionization layer by increasing the electric field. As seen in the cross-section of the model in Fig. 2.8, the normal electric field intensity between the corona and collector follows a steep decay. However, when a space charge exists, the effective electric field intensity and distance between electrodes varies with the movement of the positive space charge cloud. The red bar radial dispersion is the space charge formation between the electrodes following the associated configuration below. Essentially, the air gap size is effectively decreased while maintaining a relatively high electric field. This contributes to the overall Coulomb force affecting the particles.

![Figure 2.8. Space charge effect](image)

Figure 2.8. Space charge effect
Because this space charge exists, there are hypotheses for a minimum radius of the corona electrode at which the rate of ionization becomes saturated and optimizes the effect of the space charge [26].

2.2.6 Negative Corona

In the case of a negative electric potential applied to the corona, free electrons are repelled away from the high negative electric field and accelerate towards the grounded collector while a positive sheath of ions surrounds the negative corona. When the electrons collide with the nitrogen or oxygen molecules in the air, particle impact ionization (and the subsequent electron avalanche effect) occurs and forms positive ions in a much larger ionization layer. The free electrons with insufficient energy to cause ionization will form a less uniform boundary of the ionization layer. The electrons that drift with low speed within the drift layer attach to neutral oxygen molecules, resulting in a space charge of negative ions. This negative ion-cloud will cancel the negative corona’s electric field, therefore causing the ionization process to stop. Finally, the ion-cloud will then drift towards the positive collector electrode and transfer momentum in the same manner as with the positive corona.

In the case of a positive corona, the rate of ionization can continue as the neutral air molecules exiting the bottom of the collector produces thrust. However, the negative corona requires time for the negative space charge to drift towards the positive electrode before the electric field can recover to restart the ionization process. This recovery is time dependent and explains the observed bursts of ionization present in a negative corona [25].
2.2.6 Thrust Production Analysis

Consider the force body diagram of the ion-wind phenomenon in Fig. 2.9. The labeled forces analyzed in this diagram represent the Coulomb attractive and repulsive forces between the corona electrode, collector, and positive ion space charge.

The corona wire experiences a Coulomb repulsive force $F_{wi}$ due to the positive space charge and an attractive force $-F_f$ due to the collector foil electrode. Similarly, the positive space charge experiences a force, $F_{wi} + F_{fi}$, from the wire and foil, and the collector experiences a force, $F_w + F_{fi}$, from the wire and space charge.

![Figure 2.9. Force body diagram of Ion-wind](image)

If the electrodes and the space charge are treated together as a single system, the net force is zero. However, since the positive ions are transferring their momentum to the neutral air molecules, there is a net force consisting of the attractive and repulsive forces on the space charge cloud. Therefore, the Coulomb force acting on the ions becomes an electric body force on the air molecules and gives rise to the EHD flow. This Coulomb force is the main contribution to the ionic airflow that causes the system to levitate. The high frequency of collisions between the space charge and air bulk can be theoretically estimated as a full
transfer of momentum, which contributes to the resultant electrical body force responsible for EHD flow.

Essentially, the electric field from the corona electrode exerts a force on the neutral air molecules through the transfer of momentum from the positive ions. This, in turn, leads the equal and opposite force produced by the air molecules to push against the EHD Thrusters through the repulsive force from the ions. Therefore, an accurate measurement of the force exerted by the air exiting the collector side over a flat surface should be the same as the equal and opposite force applied to the device.

Omitted from this diagram are the other forces that act on the electrodes as a result of the ionic-wind effect – most importantly, the viscous drag and the pressure forces. These forces will reduce the force contributed by the air flow. Compared to the overall force production, the opposing pressure forces are negligible as observed in the simulations (See Section 7).

Since this EHD flow accounts for most of the force driving the thruster, the magnitude of the force generated mainly depends on the accelerating voltage gradient (which accelerates the positive ions between each impact) and the volume of neutral molecules (which receives the momentum exchange in the downward, vertical direction). Geometric factors (e.g., the radius of curvature of the collector electrode, the radius of curvature of the corona electrode, and the distance between electrodes) govern the optimal electric field and momentum transfer. The subsequent sections will discuss the physical limitations and electrical properties mentioned in this detailed explanation that control the boundary conditions to the specific design parameters of the thruster.
2.3 Corona Discharge

If the voltage is gradually increased above a certain critical value (CIV) between two parallel wires placed a distance apart, a visual corona will appear as the first evidence of stress in the air [25]. This glow begins near the conductor’s surface because the dielectric flux density is greatest there. If the voltage is further increased, the wire glows brighter and the corona electrode will have the appearance of extending farther from its surface; this is due to the presence of an ionization layer. Due to the physical properties of air, an electric field with a voltage that exceeds air breakdown will cause a spark to bridge between the electrodes. Specifying the ratio between the inter-electrode distance, or air gap, and the radius of the wire is important because a critical value exists where a spark that bridges the electrodes can occur before the corona discharge phenomenon appears [2].

In the study of EHD phenomena, such as ion-wind or thrust, it is important to understand the cause of corona discharge, how it is different from electrical arcing, and how it effects EHD Thruster operation. A corona is formed when the high electric field around an electrode of low curvature ionizes the air (or other neutral fluid) around the electrode. The ions flow away from the corona toward a region of opposite charge and eventually neutralize. When a corona forms around a sharp electrode, the conductive ions extend the effective area of the electrode. In this way, the outside of the corona region becomes the effective surface of the electrode. Because the curvature of the corona will become larger as the corona extends further from the actual electrode, corona generation stops at the distance where the effective electrode curvature (and electric field strength) is not sufficient to ionize air [27]. The size of conductor is increased by the conducting corona, which therefore increases until the flux density is below the threshold gradient before breakdown. It has been experimentally
shown that the gradient at the breakdown of the conductor surface increases with decreasing diameters of conductors.

More concretely, corona discharge is a low energy electrical discharge with non-thermal ionization that is sustained with no external energy other than the applied potential difference. This should not be mistaken with an electric arc, which is a disruption of a gaseous dielectric (in this case air) from one electrode to another electrode [28]. This phenomenon occurs when the voltage at the wire surface just exceeds the breakdown strength of air. The air at this point becomes conductive, therefore increasing the effective size of the conductor. If the increase in size of the conductor by the conducting air increases the gradient, the broken-down area will continue to enlarge and cause electric arcing. This electric arcing is more likely to occur at corners where there is non-uniform distribution (as seen in Fig 2.10). This is essentially a short circuit, which leads to the high current, high power discharge, and high heat and luminescence. Corona discharge, on the other hand, can be sustained for a longer period of time with a low current, low intensity photon emissions, and low energy dissipation, which can produce low density ionization with only a few mW of power [28]. It has been experimentally proven that in parallel wires, corona does not form when $d/r < 5.85$, where $d$ is the spacing between the center of the conductor and $r$ is the conductor radius [25].
As discussed in Section 2.2.6, corona discharge occurs with positive and negative corona electrodes. A corona generated by a positive electrode is referred to as a positive corona, and a corona generated by a negative electrode is known as a negative corona. Positive and negative coronas have different characteristics because of the different types of particles involved in the mechanism. In the case of a positive corona, positive air ions make up the corona. On the other hand, with a negative corona, electrons are responsible for the corona. This difference is important for calculating power loss in an EHD Thruster, which is discussed in the next section.

2.4 Electrode Polarity

An EHD Thruster will generate thrust with a positive corona electrode (negative collector) and with a negative corona electrode (positive collector). It does not matter whether the corona electrode is positive or negative. Even though the ionization and ion acceleration effect and theory are virtually the same, there are factors that make positive corona electrode EHD Thrusters much more common than negative corona electrode EHD
Thrusters. One of the more important considerations is power loss. Power loss in an EHD Thruster is dependent on the voltage across the corona generation region (or the area around the corona electrode). The corona generation region is described by an effective wire radius, \( r_0 \) [29]. Corona discharge around the wire effectively extends the wire’s conduction region as described in the previous section. Thus, the wire behaves as if it has a larger radius. Fig. 2.11 illustrates the difference between the actual wire radius and the effective wire radius.

The corona voltage drop is described by:

\[
V = E_t r \ln \left( \frac{r_0}{r_w} \right),
\]

(2.1)

where \( E_t \) is the electric field at the corona electrode necessary for corona discharge, and \( r_w \) is the actual electrode radius [29].

It has been shown that for a negative corona electrode, the effective wire radius is approximately twice that for a positive corona electrode [29]. Thus, from the equation above, it is clear that there exists a greater voltage drop and power loss when using a negative corona electrode. This is one of the main reasons for using a positive corona electrode as opposed to a negative electrode.
Corona can also be generated by an AC sinusoidal or pulsed potential [31]. The duty cycle of the waveform can be modified to reduce power consumption or increase the rate of ionization. Alternating high voltage in a controllable manner is difficult, and has been avoided by most studies involving EHD Thrusters, but more closely pursued for industrial purposes, such as power discharge systems [32].

### 2.5 Air Breakdown

The conditions under which air breaks down and becomes a conductor are important for EHD Thrusters because they determine the design of the minimum corona electrode/collector gap or the maximum applied voltage. Dry, unionized air begins to conduct current at an electric field strength of approximately 30kV/cm [24]. While this value is a good starting point, it does not account for the unique conditions of the EHD Thruster.

Practically, the EHD Thruster will not be operating in absolutely dry air. Instead, it will need to operate in air where the humidity varies over a large range. More importantly, the air between the corona electrode and the collector is ionized, which significantly lowers the electric field strength necessary to cause breakdown of the air [30]. Furthermore, the electric field between the corona electrode and the collector is non-uniform. The non-uniform field is essential to the operation of the EHD Thruster.

The three unique conditions mentioned above make the 30kV/cm value for air breakdown inaccurate. Using the proof-of-concept design discussed in later sections, a breakdown electric field strength of approximately 8.8kV/cm was measured. While this experimentally determined value will vary based on humidity and specifications of the EHD Thruster, it is a better approximation for the air breakdown voltage than the 30kV/cm value.
2.6 Corona Inception Voltage (CIV)

The positive corona electrode must exceed a certain potential in order to create an electric field capable of ionizing air. In other words, the voltage must be high enough to create corona discharge, an electric discharge due to the ionization of air. At this potential, the electric field is not quite strong enough to cause air breakdown, but maintains a threshold that creates the flow of ions in the drift layer. Above this potential, an Ohm’s law regime exists that follows a proportionate current to voltage increase. After this region is the breakdown region where the current increases more rapidly and leads to the electrical arcing. Below this minimum potential, the amount of ionized air – and therefore the thrust produced by the EHD device – is approximately zero [33].

The CIV can be calculated using Peek’s equation\(^2\), which determines the voltage necessary to produce corona discharge [33].

\[
CIV = E_i r \ln \left( \frac{d}{r} \right) \quad (2.2)
\]

\[
E_i = E_0 \delta \left( 1 + \frac{Y}{\sqrt{r}} \right) \quad (2.3)
\]

\(E_i\): Electric field strength necessary to produce corona discharge (V/m)

\(E_0\): Electric field strength necessary to break down air (V/m)

\(\gamma\): Peek’s value uses \(0.301 \sqrt{\text{cm}}\), but convert to V/m to maintain consistent SI units

\[
(1\text{cm})^{-\frac{1}{2}} \times \left( \frac{1\text{m}}{100\text{cm}} \right)^{-\frac{1}{2}} = 0.1 \Rightarrow 0.0301 \sqrt{\text{m}}
\]

\(r\): Radius of positive corona electrode (m)

\(^2\) The original empirical formulation from Peek’s textbook [33] uses kV/cm, but recent literature converts the formula to maintain consistent SI units.
\(d\): Distance between the positive and negative electrodes, or air gap (m)

\(\delta\): Air density factor, where:

\[
\delta = \frac{3.92 \ast (\text{barometric pressure in cm})}{(273 + \text{temperature in } ^\circ\text{C})}
\]

At STP, temperature is 25 °C and pressure is 76 cm. Therefore, \(\delta = 1\).

Notice that Peek’s equation depends heavily on the geometry. Electric field intensity is greater around the surface of a charged conductor with a lower radius of curvature. This follows the basic electric field intensity formula of a sphere with charge \(Q\), given by:

\[
E = \frac{Q}{4\pi \varepsilon_0 r}
\]

Therefore, as the radius decreases, the electric field increases.

### 2.7 Ionization Layer

The ionization layer is the plasma-extension of the corona electrode radius during corona discharge. Ideally, this volume only consists of free positive ions and electrons as a direct result from impact ionization between the free electrons and neutral molecules.

This ionization layer maintains a high mean kinetic energy, and follows properties in the plasma state. Approximately 99% of the space is in the plasma state due to the abundance of solar and stellar matter. These plasmas are classified as high temperature plasmas, which have high densities (on the order of \(10^8\) particles cm\(^{-3}\)) at temperatures exceeding \(10^6\)K [23]. In contrast, the plasma generated within the ionization layer has a very low density (on the order of \(10^6\) particles cm\(^{-3}\)) at low temperatures in the region of 300K. Due to the charge carriers within the ionized gaseous mixture, plasma acts as a conductive fluid and is strongly influenced by the effect of electric and magnetic fields described in
Maxwell’s equations. In this study, the electrical current produced in the corona discharge is small enough to ignore magnetic induction.

After the ionization layer is formed, certain electrical properties are assumed to simplify our analysis. Kaptsov’s assumption states that the electric field increases proportionally to the voltage below the CIV, but will preserve its value after the corona discharge is initiated [34]. This means that the potential at the corona and the boundary of the ionization layer will stay constant when the CIV is reached. In addition, corona discharge is assumed to be uniformly distributed over the surface of the corona electrode.

Peek’s equation, as discussed earlier, solves for the voltage levels at the electrodes, and was used to derive important formulas for solving the boundary conditions of our governing equations.
Chapter 3:

EHD Thruster Derivation

In this chapter, we will discuss three mathematical models used to account for the thrust production of the EHD Thruster phenomenon. Each model was hypothesized, tested, and refined to solve for an approximation of the force produced by the EHD phenomenon. The ion momentum transfer model refers to the transfer of momentum based on the drift velocity of ions. However, the force calculated by this method, (applied under typical experimental conditions) is two orders of magnitude too small. Thus, an extension to the ion momentum transfer model was designed to focus on the group of ions’ continuous electrical relationship with the electrical field of the collector electrode in a space charge cloud formation. This model is known as the Electro-hydrodynamic (EHD) flow model. Expanding upon this model specifically for thruster design principles, Evgenij Barsoukov formulated a refined set of equations that relates the force produced to major geometric and power variables, such as voltage, radius of curvature, length, and air gap.

3.1 Ion momentum transfer model

The initial theoretical explanation proposes an ionic-wind effect resulting from the transfer of momentum between the positive ions and neutral molecules. Although these
events occur in the manner discussed in earlier sections, a complete dependence on ion momentum transfer is two orders of magnitude below the observed experimental thrust production.

The ion momentum transfer model approaches the problem by finding the force produced when all of the kinetic energy of the ions is transferred to momentum. Starting with the kinetic energy equation, \( \frac{1}{2} m v^2 = qV \), we substitute the drift velocity, \( v_d = \frac{F q}{I m} \), calculated from the rate of change of momentum equation, \( F = m v_d \frac{I}{q} \). The resulting force and voltage equation is represented as

\[
\frac{1}{2} m \left( \frac{F q}{I m} \right)^2 = qV
\]

\[
F(I, m, q, V) = I \left( \frac{2mV}{q} \right)^{1/2}
\]

(3.1)

where \( F(I, m, q, V) \) is the force felt by the charged particle, which is dependent on the current \( (I) \), mass of the particle \( (m) \), charge of the particle \( (q) \), and voltage between electrodes \( (V) \).

To check the validity of this equation, we’ll substitute \( F = \frac{M}{g} \) and perform a dimensional analysis approximation with some typical values for the voltage and current used from experiments. Assuming that copper ions are stripped from the corona wire during ionization, we can use this heavier particle as a reference.

\[
M = \frac{I}{g} \left( \frac{2mV}{q} \right)^{1/2} = \left( \frac{1mA}{9.81 \frac{m}{s^2}} \right) \left( \frac{(2)(63.55)(1.66 \times 10^{-27} \text{kg})(20kV)}{1.6 \times 10^{-19} \text{C}} \right)^{1/2} = 0.0167g
\]

This means that the complete transfer of momentum, assuming the heavier particle reference, will still yield a force that is two orders of magnitude smaller than the weight of an
average lifter (~3g). Obviously, this approach is an incomplete representation of the ionic-wind production; therefore other models were explored to explain the missing force.

3.2 EHD Flow model

One of the major problems with the ionic wind approach is the lack of consideration for the electric field forces. The effect of an ionic wind may be the underlying force production, but the additional thrust depends on ion mobility. With the EHD flow approach, we try to calculate the force felt by the collector from the corona discharge at a distance of the air gap. The force should be independent of the actual ion path and electric field. It represents the momentum transfer rate between the space charge ion-cloud to the neutral air molecules, and can be used to calculate the gas flow and velocity exiting the grid.

To derive the widely accepted EHD model [35], we must replace the charge represented by the kinetic energy equation with a generic charge for the particle. This leads to a charge distribution as a function of the distance between the electrodes as,

\[ \frac{q}{d} = \frac{I}{v_d} \]

where \( v_d \) is the drift velocity (average speed at which the ions travel the distance, \( d \)) and \( I \) is current. The drift velocity of the ions influenced by the electric field is defined with the ion’s mobility, \( \mu_i \), to obtain the following relationship:

\[ v_d = \mu_i E = \frac{\mu_i V}{d} \]

where \( V \) is the voltage between electrodes \( d \) is the distance between electrodes (air gap).

From these equations, a thrust to current relationship is derived as follows:
\[ F = q E = \frac{q V}{d} = \left( \frac{I}{\nu_d} \right) \left( \frac{V}{d} \right) = \left( \frac{I}{\mu_i V} \right) \left( \frac{V}{d} \right) \]

\[ \therefore F = \frac{I d}{\mu_i} \]  

(3.2)

In equation (3.2), \( F \) is the force in Newtons, \( I \) is current in Amperes, \( d \) is the air gap in meters, and \( \mu_i \) is the ion mobility. It is interesting to note that negative ions are faster than the positive ions, but will cause less force because force is inversely proportional to the ion mobility. For reference, the ion mobility values for negative and positive ions are \( \mu_n = 2.7 \times 10^{-4} \, \text{m}^2/\text{Vs} \) and \( \mu_p = 2.0 \times 10^{-4} \, \text{m}^2/\text{Vs} \). The EHD model assumes that all the electric energy of the field is ultimately transferred to neutral molecules. Essentially, the force along all the field lines represented by the current was projected over the plane grid.

Similar to the sanity-check approximation performed for the ion momentum transfer model, the EHD Flow equation was substituted with some typical experimental values. Since air consists of approximately 78% nitrogen, it is valid for the approximation to use the nitrogen ion mobility value for the equation.

\[ F = \frac{I d}{\mu_i} = \frac{(1 \text{mA})(0.04 \text{m})}{(2.5 \text{cm}^2/\text{Vs}) (10^{-2} \text{ m/cm})^2} = 0.16 \text{ N} \]

\[ M = \frac{F}{g} = \frac{0.16 \text{N}}{9.81 \text{m/s}^2} = 16.3 \text{ g} \]

Working out the math above, we see that 16.3g can be theoretically lifted by the use of nitrogen ions following the EHD flow principle. Although this approximation assumes the current is entirely due to the nitrogen ion mobility, it is comforting to know that the thrust production is a reasonable order of magnitude to explain the phenomenon.
If we account for the total force of the EHD Thruster system, there exists a negative force due to the loss of momentum when the ions hit the collector. The equation

\[ F_{total} = \frac{I d}{\mu_i} - F_{lost} \]

is discussed in [30], however, the contribution of this “frictional force,” \( F_{lost} \), is small compared to the total force and ultimately neglected. At high force values, this viscous drag may have an effect, and the simulation of the force production takes this into account.

The EHD model’s force is now represented as a function of air gap, current, and ion mobility; but, how does one design with current? Knowing the current does not directly relate to the adjustable parameters in the anatomy of the EHD Thruster. To better understand the voltage-current relationship, we look at Barsoukov’s model.

### 3.3 Barsoukov’s model

Evgenij Barsoukov [30] developed a model to explain the current flow (I) in the previous EHD model by utilizing existing approximations with Electrostatic Precipitators (ESPs). This combination of concepts yielded a thrust production formula with geometric thruster design parameters instead of current values. The following voltage-current characteristic between a wire and plate configuration (derived from Cooperman’s model for ESPs [36]) was used to relate the current to the voltage:

\[ I = \mu_i GV (V - CIV) \]  \hspace{1cm} (3.3)

In equation (3.3), \( G \) is a geometric factor that relates the wire radius (\( r \)), width of plate (\( W \)), and length of collector (\( L \)) for a two-electrode configuration.
Barsoukov’s original model considered a flat collector surface; however, in our case, it is more accurate to represent a rounded collector surface. The width of the collector is the length of the exposed surface of the corona wire, which is half of the circumference of the collector.

\[
W = \frac{\text{Collector}_\text{circumference}}{2} = \frac{2\pi r_c}{2} = \pi r_c
\]

After substituting this value into Cooperman’s equations (3.3, 3.4, and 3.5), the resulting equation yields a slightly modified version of Barsoukov’s model relating thrust production to adjustable physical variables:

\[
F(V, L, d, r_w, r_c) = 2\pi \varepsilon_0 LV \frac{V - E_0 \delta r_w \ln \left( \frac{d}{r_w} \right) \left( 1 + \frac{0.0301}{\sqrt{r_w}} \right)}{d \ln \left( \frac{\pi^2 r_c e^d/\tau_c}{2 r_w} \right)}
\]

Notice that the force is proportional to approximately \(V^2\). The increase of voltage leads to an increase in ion generation. Each ion then contributes to the space charge density and the electron avalanche effect, which follows a polynomial direct relationship. From equation (3.5), the force increases when: the voltage increases, length increases, wire radius decreases (subtracting less from \(V\)), air gap decreases (at a rate following \(\frac{\ln(d)}{d}\)), or the foil
radius decreases (following approximately $\frac{1}{\ln (r_e)}$). Understanding these relationships is essential for the design process and testing phases of the experiments.

To test the validity of this formula, the experimental values of the single thruster tests from the experiments conducted during Senior Projects (further explained in Chapter 5) were substituted to obtain:

$$F(2.5 \times 10^4 V, 0.2m, 0.025m, 4.5 \times 10^{-5}m, 1 \times 10^{-3}m) = 0.0076N$$

The result of the experimental measurements (conducted during Senior Projects) of 0.0062N was surprisingly close in comparison to the theoretical prediction. Other experimental values from the single thruster variable tests compared to the equation’s solution within a +/- 0.001N deviation. The largest discrepancies between these values existed near higher voltages where premature air breakdown and viscous drag forces on the collector reduced the force values in experimental results. In addition, the stronger electric fields due to the high voltages may have caused some electrical discrepancies in our measurement methods. These methods, along with the experiments conducted throughout this thesis, will be explained in Chapter 5.

Barsoukov’s model is the current EHD Thruster theory used to calculate the thrust generated in a more general sense, but advancements in simulation technologies have changed the usage of this equation for approximations and sanity-checks for results and design purposes. The governing differential equations used for the EHD Thruster simulations discussed in Chapter 6 are the more common method of approximating thrust production.
Chapter 4:

EHD Thruster Design

This chapter will combine the theory from the previous sections with practical design parameters to obtain the dimensions for the proof-of-concept flight. Barsoukov’s model (equation 3.6) can be analyzed to conclude many thruster design assumptions that confirm the electrical theory and offer general guidelines for maximum thrust production. The main variables that can be adjusted in the EHD Thruster design are the applied voltage, radius of the wire, length of the thruster, radius of the collector, air gap, materials selection, and overall thruster geometry. The adjustment of these variables will be discussed in detail in relation to obtaining maximum thrust. For most cases, adjusting each variable will cause an offset to other variables. This leads to major design tradeoffs when analyzing the ideal values for the radius of curvature of the collector, thruster geometry, scaling issues, and air gap. These design properties provide the fundamental advantages and disadvantages of specific thruster orientations, which will be valuable when testing different approaches in Chapter 5.
4.1 Following Barsoukov’s Model

The main route to increase the thrust is to maximize the applied voltage to the corona electrode bounded by the air breakdown voltage. As discussed in the theoretical section, this air breakdown voltage level is dependent on: the electric potential distribution in the drift layer (which is directly affected by the air gap and radius of curvature), and the physical properties of the dielectric medium. As seen in Fig. 4.1, if the corona wire and collector foil are not parallel, there exists a non-uniform electric field distribution that can cause premature air breakdown at the closer point. When designing the thruster, it is important to maintain a taut wire to prevent this event.

![Figure 4.1. Non-uniform distribution of electric field](image)

The radius of curvature of the corona wire should be decreased to increase the electric field and increase ionization. Individual strands from disassembled stranded-wire, thin copper wire, and magnet wire were tested due to their conductivity properties, availability in the lab, and small radius of curvature. In addition, a rough surface on the corona wire increases the number of sharp edges and leads to a lower CIV. Remember that the existence of the space charge shows that there is a bounding limit to the overall effectiveness of this
optimization. The minimum radius of curvature of the collector can be determined with Peek’s equation and will be discussed in Section 4.4.

Obviously, a longer thruster would provide more neutral air molecules (which in this case, is used as the fuel), but more supports are required to maintain the thruster size. This introduces a question about scalability with EHD Thrusters that has yet to be solved. The size of the thruster can increase in size, but the weight of the materials required for maintaining the structural stability is questionable. The complexities and tradeoffs of the air gap which determines the maximum thrust that can be produced before air breakdown will be discussed in Section 4.3.

### 4.2 EHD Thruster Geometry

The ideal shape of an EHD Thruster is a circle. By eliminating all corners from the shape of the corona electrode and collector, the electric field is constant everywhere on the thruster [37]. If the thruster shape has sharp corners, there will be localized areas of higher electric field near the edges of the thruster. Consequently, the areas of higher electric field strength will have a greater chance of exceeding the air breakdown voltage and creating a short circuit when the voltage is increased between the electrodes. In addition, the higher field intensity leads to an increase in the ionization of air molecules on the collector electrode, which cancels a portion of the device’s thrust. Theoretically, the use of a circular thruster design allows for the maximum voltage to be applied for a particular air gap. Consequently, maximum thrust can be extracted from the design.

A polygon thruster design offers many of the advantages of a circular design but is more practical to construct using available lightweight materials. Since the corners in a
hexagon, for instance, are not particularly sharp, the electric field should be relatively uniform over the entire thruster. An important concept to realize when choosing the number of sides to use in a polygon thruster is the weight of the structure. For each corner added in a polygon design, there must be a support that holds the corona electrode directly above the collector. If there are many corners, then many supports are necessary, which increases weight significantly. Thus, there is a tradeoff between electric field uniformity and thruster weight. Obtaining a uniform electric field between the corona electrode and collector throughout the entire thruster will increase thruster weight because more supports are necessary in the structure.

Once a basic thruster shape is chosen, there are numerous techniques available for increasing the force that the EHD Thruster can generate. Stacking and nesting techniques have been used by multiple researchers to increase thrust and lift a larger payload [27, 36]. The objective of stacking or nesting thruster stages is to multiply total thrust while increasing thrust-per-weight or thrust-per-unit-area. It is possible to increase thrust-per-unit-area because several thrusters can share the same structure when they are stacked or nested. Thus, for example, the weight of an EHD Thruster with five nested components is less than the weight of five individual thrusters. However, the nested design makes just as much thrust as the combination of the five individual thrusters.

Unfortunately, the design of a nested or stacked EHD Thruster is more complex than the design of a single thruster. It is necessary to accurately calculate the spacing between nested thruster stages to ensure that the field distortion between different stages is not significant enough to cause premature breakdown or irregular airflow. Such an analysis requiring simulation software will be explored with COMSOL Multiphysics.
4.3 Electrode-Collector Air Gap

Since we were unsure of the electric field strength necessary to break down air under the operating conditions present in the EHD Thruster, the air gap between the corona electrode and the collector was designed to be adjustable up to 5 cm. Once an approximate breakdown electric field strength was measured for the initial proof-of-concept design, then this value could be used to choose an air gap that matched the voltage applied to the EHD Thruster.

The air gap for the hexagonal design could be adjusted by making new holes in the balsa wood posts at each corner of the thruster and passing the corona electrode wire through the holes.

![Graphs showing force vs air gap](image)

Figure 4.2. (Left) Force vs air gap for increasing breakdown voltages. (Right) Force vs air gap for a fixed voltage.

Adjusting the distance of the air gap is a design tradeoff between the window of operation and onset voltage. Increasing the air gap will increase the size of the drift layer, thereby introducing more neutral molecules to cause more collisions allowing for a higher maximum thrust. Fig. 4.2 (left) shows the measured force to air gap direct-relationship from
1 to 5 cm separations set to 0.5 cm increments when the breakdown voltage is reached for each air gap experiment. By allowing more space between electrodes, a higher applied voltage and larger thrust window of operability can be obtained. However, increasing the air gap also leads to a weaker electric field, and thus requires a higher CIV to create corona discharge. Although a higher force can be reached with a larger air gap, there is a lower force when comparing specific voltages of smaller air gaps.

If the voltage were fixed as the air gap was altered, the graph would follow Fig. 4.2 (right), which shows that increasing the air gap with a fixed voltage will decay following \( \ln\left(\frac{d}{a}\right) \) curve. The major tradeoff when designing the air gap involves the initial CIV for the window of operation and limitations given by the power supply. If any maximum voltage could be applied, the largest force would be obtained with the largest air gap with a correspondingly high voltage at CIV. When you increase the size of the air gap the drift layer size increases and provides more positive ions to contribute to the space charge forces.

### 4.4 Collector Curvature

From a design perspective, the radius of collector should be larger than the radius of the corona electrode in order to never ionize air at the CIV. If the collector begins to ionize the air, the opposite ions created would counteract the thrust created by the intended corona through reduced ion attraction and a corona discharge created at the collector [27]. The radius of curvature of the collector must be large enough to prevent corona discharge and can be determined by Peek’s equation. As a general order of magnitude, the radius of the corona wire used was 0.045 mm, and the collector curvature was designed to be 1.7 mm. However,
we cannot simply set this arbitrarily to a large radius because the neutral molecules will impact the collector surface and push the thruster downwards, thereby counteracting the force achieved from the space charge.

Therefore, the CIV for the negative electrode must be higher than the maximum voltage that will be applied between the electrodes. Consequently, no corona will be generated on the collector for all possible thruster operating conditions. Air breakdown should occur before the CIV for the collector is reached. We need to solve Peek’s equation for \( r_c \) in order to find the minimum collector radius.

\[
CIV = E_0 \delta r_c \ln \left( \frac{d}{r_c} \right) \left( 1 + \frac{0.0301}{\sqrt{r_c}} \right)
\]

The size of the collector electrode is a compromise. If the collector has a large radius, it will be very unlikely to create a corona discharge. However, there will be more area for air molecules to impact as they are being forced through the bottom of the EHD Thruster by the electric field. If the radius of the collector is too small, air ionization will occur at the collector similar to the effect at the corona electrode. This forces electrons upwards, which neutralizes thrust-producing positive air ions. By applying the same logic as with designing the collector, we know that we want the corona electrode to have the smallest possible radius. As a result, corona generation will occur at a lower voltage allowing for less power consumption. A fine piece of inductor coil wire with a diameter of 45\( \mu \)m was used for the corona.

Using Peek’s equation, an expression for the minimum collector radius as a function of applied thruster voltage and corona electrode radius can be obtained [27].
In the above equation, \( r_c \) represents the collector radius in millimeters, \( r_w \) is the corona electrode wire radius in meters, and \( V \) is the applied voltage in kilovolts. Using equation (4.1) with an applied voltage of 20 kV (the maximum voltage of the power supply we were using at the time) and a wire radius of 45\( \mu \)m (the thinnest wire that we could attain), the minimum collector curvature for the initial proof-of-concept design was calculated to be 1.22mm. To ensure that corona inception would not occur at the collector, the radius was made slightly larger than calculated. The actual collector had a radius of about 1.5mm, which would allow the collector to resist corona inception up to 25kV of applied voltage.

4.5 Power Consumption

The electrons moving toward the positive corona electrode and the positive ions moving toward the negative collector in an EHD Thruster contribute to a current flowing from the corona electrode to the collector. A longer length of corona electrode will ionize more air and therefore, create more current flow. By measuring current flow during thruster operation \( I \) and knowing the applied voltage \( V \), power consumption can be calculated with the following equation:

\[
P_{max} = I_{max} \times V_{max}
\]

The calculation of power consumption is relatively simple in the case of EHD propulsion since the thruster is a dc device requiring a steady current and voltage to function properly.
4.6 Thruster Design Summary

The design of an EHD Thruster has major tradeoffs including the collector radius, thruster geometry, and scalability issues. Increasing the radius of curvature of the collector will allow a higher applied voltage before an ionization layer forms around the collector. If ionization occurs at the collector, there will be an undesired cancelation in thrust. On the other hand, if the collector was a simple plate design, there will be increased air impact, which pushes the system downwards.

The overall thruster scalability issue is difficult to address with the large dimensionality of variables that effect the overall thrust production. Increasing the length of the overall EHD Thrusters will increase ionization and thrust, but the extra weight required to support the structure is variable. In addition, the larger length of corona wire will require more time for the electric field to distribute, and therefore cause an undesired non-uniform effect. This non-uniformity could cause premature air breakdown or overall non-uniform thrust.

The geometry of the thruster is important to maintain a uniform electric field to prevent premature air breakdown. This can be achieved by increasing the number of corners in the geometry. However, additional corners require more structural support, which increases the weight of the thruster. As seen in the low thrust production approximated by Barsoukov’s model in Chapter 3 suggests that weight reduction is the most important variable to consider in order for the device to achieve lift.

Based on the availability and lightweight properties of the considered materials, balsa wood, aluminum foil, and different calibers of wire were used as the primary materials to construct the proof-of-concept design discussed in the next chapter. Using the hypotheses
and conclusions from this chapter, a theoretical proof-of-concept thruster was constructed and tested during Senior Projects.
Chapter 5:

EHD Thruster Experiments

The results of the research conducted to derive electrical concepts, theoretical explanations, mathematical models, and design parameters, was applied to a proof-of-concept EHD Thruster design, built and tested for flight during Senior Projects (with Miroslav Ovcharik as my Senior Project partner). Through the frustrations of converting theoretically-sound proofs to real-life working models, many tweaks were observed and will be shared for reference in improving future models.

The successful flight of the EHD Triangle Lifter from the previous year led to experiments involving more specific variations in the controllable variables, such as air gap, materials for corona wire, materials for collector, and collector shape. A single EHD Thruster structural framework was built specifically to vary these parameters. Measurements of thrust production and model thrusters utilizing these optimized values were tested to improve lift and control.

All of the physical experiments were conducted during Senior Projects, but the numeric results from the single thruster frame are relevant for the main comparisons to the COMSOL Multiphysics simulation predictions (Chapter 7). Given the more in-depth theoretical analysis this year, the experimental conclusions from Senior Projects were refined and will be reiterated in this Chapter for clarity. This Master’s thesis is the theoretical
continuation of the Senior Project thesis, but with a closer focus on parameter optimization rather than achieving EHD thruster flight.

5.1 Safety Considerations

As a general warning, before performing any experiments involving high voltage, it is very important that the people conducting the experiment (and people that may come in contact with this laboratory area) are aware of the safety instructions. An instructor was required to be present during these tests and all equipment was sufficiently secured after usage.

5.1.1 High Voltage and Experimental Setup

High voltages can be very dangerous and can cause severe injury if not used with precautions. The lab bench setup for our tests was properly quarantined with a Plexiglas encasing, and warning signs of high voltage usage were posted around the area. When the digital high voltage power supply (from Matsusada Inc, 35kV, 2mA) was not in use, a lock was placed on the plug to prevent unauthorized usage. Safety equipment, such as high voltage rubber gloves, special boots, and a full apron shield, were used when operating the power supply. Electronic devices, jewelry, and other metal objects were removed from the vicinity during usage because the generated electric field could still be damaging without direct contact.
When the power supply is turned off, the metal surfaces may still be charged. Thus, they should be safely discharged by connecting a resistor between the positive and negative terminal. The warning precautions and first-aid procedures included with the power supply were placed in the area. Our specific lab bench was organized such that wires did not touch the ground and the output leads were arranged far apart (and attached to a solid structure) to prevent short circuits from exposed terminals.

5.1.2 Ozone Creation

Whenever corona discharge is present, there is always the characteristic odor of ozone. Air consists of a mixture of oxygen (O₂) and nitrogen (N). A high voltage gradient in the corona region breaks the two oxygen atoms apart. The single oxygen atom is chemically active and recombines to form the O₂ and O₃ ozone molecules. The oxygen molecules in their ionic O state also combine with metal or organic matter to form oxides. The oxides formed under these conditions are hazardous to health at certain levels [38]. Thus, the energy loss by the corona manifests as heat, chemical action, light, noise, and convection.
The oxides formed by ozone can cause undesirable physiological effects on the central nervous system, heart, and vision. Symptoms may include irritation to the lungs, leading to pulmonary edema [39]. Normal DC corona operating in air generates toxic ozone and other ions, which do not decay. Depending on exposure time, this health hazard can be cumulative. 0.1 PPM is the maximum allowable safe concentration of ozone for an 8-hour industrial exposure (from the National Ambient Air Quality Standards). To reduce the health risks to the people within and around the laboratory, the test area was well-ventilated and experiments did not exceed 30-minute periods.

These rules may have been inconvenient, but they existed for the protection and well-being of the students at The Cooper Union. Only under these conditions were the EHD Lifters and test devices allowed to operate.

5.2 Experimental Setup

The EHD Lifters in Fig. 5.2 were built with the overall objective to maximize the thrust-per-unit-weight by testing hypotheses regarding thruster geometry and materials. Preliminary research of EHD physics and existing EHD experiments [37] provided a starting point for the initial design parameters, including collector curvature, air gap distance between electrode and collector, and foil height (discussed in previous Chapters). The initial design was made from balsa wood, cheap aluminum foil, and a single strand of stranded electrical wire. Initial EHD lifters followed the theoretical conclusions mentioned in Chapter 4, including the hexagonal shape which balances the structural support weight while maintaining a relatively uniform electric field distribution along the collector. This initial
design was too heavy and did not lift. Promisingly, the hissing sound due to the air moving past the corona electrode and foil collector indicated ionic wind generation.

Figure 5.2. Experiments leading to proof-of-concept flight

The latter designs from the Senior Project experiments focused on reducing the weight by utilizing minimal structural supports. To further reduce the weight, a collector design utilizing foil without a balsa wood frame was constructed. The foil itself was the structure for the thruster. Although the frame was very lightweight, the combination of malleable aluminum foil and wire was difficult to mold to maintain an equal air gap distance. The variation in the air gap between the corona electrode and collector led to localized areas of increased electric field. The electric field was high in spots where the air gap was small and too low in places where the air gap was higher. The result was that air breakdown occurred at the points where the air gap was smallest. If the voltage was decreased to avoid air breakdown, the thrust was too low and was unevenly distributed. Overall, this design helped reduce weight significantly but was impractical due to its low structural rigidity, which led to large variations in air gap between the corona electrode and collector around the perimeter of the thruster. Without the structural support of the balsa wood, it was very difficult to keep the wire taut and directly on the same plane as the collector surface.
Conductive foam was considered as a material for the collector surface due to its high rigidity compared to the aluminum foil. A conductive foam sheet was measured to have a density of approximately 150.3 g/m$^2$, and the aluminum foil has a density of 45 g/m$^2$. Although the foam is heavier, its rigidity allows for it to be used as the structure of the thruster, which makes it lighter than a balsa wood frame with an aluminum foil collector. The foam is rigid enough to properly support the corona electrode wire above the collector to ensure an even air gap. However, when tested, it was found that the foam could not interact with the ions produced by the EHD thrust mechanism.

The next design used the circular structure of a Styrofoam cup to support the corona electrode and collector of the EHD Thruster. The advantages of such a design are geometry and low weight. As described previously, the ideal geometry for an EHD Thruster is a circle since it ensures a uniform electric field throughout the structure. Using the Styrofoam as a structure resulted in a total thruster weight of only 1.9g. However, it was difficult to wrap the Styrofoam cup with a conductive collector material (such as aluminum foil) without creating sharp edges or bends in the material. The lack of a smooth collector surface caused a higher electric field and therefore premature air breakdown. When voltage was applied, there were sparks from the uneven distribution.

In addition, a wire only acts as an effective corona electrode when it is taut and directly above the collector. A taut wire increases the uniformity of the electric field and therefore ionization. The removal of a “z-offset” in terms of the electrode aligned directly above the collector forces the neutral molecules to follow the intended aerodynamic shape of the collector. The concept of a circular foil theoretically optimizes performance, but the physical construction of a smooth collector surface is very difficult.
The next design tested during senior year was a lightweight triangle balsa wood frame with a magnet wire. The magnet wire was thinner, more malleable and more conductive than the single strand from the stranded wire. The smaller radius, lighter weight, and flexible attributes of this material make it a desirable emitter. The operational proof-of-concept triangle design test model weighed 0.71g.

After constructing a working proof-of-concept model and narrowing down different design considerations, the next step was to set up a controlled experiment to determine the parameters that maximize thrust on a single thruster, such as corona electrode size, air gap variation, collector material, and collector shapes. A single thruster test removes the high electric field interactions between adjacent thrusters. In Fig. 5.3, the single thruster structure is placed upside-down on the scale to measure the air pushing downwards against the scale. The frame was designed to maintain structural integrity and support adjustable foil heights, collector designs, and electrode-collector air gap distances. The Styrofoam plates (acting as to increase the distance) and aluminum foil covering the scale (acting as a Faraday Cage) were required to prevent the electrical interference of the high electric fields.

Figure 5.3. Single thruster experiment

51
In all of the single thruster experiments, the force on the scale and current in the system were recorded while the voltage was increased at small increments. These readings continued until the first sign of electrical arcing occurred – measurements passed this point would be dangerous for usage and greatly increase the ozone creation levels.

### 5.3 Experimental Results

Using the magnet wire for the corona electrode, the air gap was varied to obtain the distance that yielded the highest thrust within the reasonable boundaries of voltage operation. Given this optimal air gap, the collector material and collector shapes were then varied for thrust production comparisons.

![Figure 5.3. Force vs Voltage focus on quadratic relationship](image)

Fig. 5.3 shows the Force vs. Voltage relationship between different collector shapes. The relationship does not follow a linear equation, but instead follows a quadratic curve. This can be explained by the combination of an increase in the volume and velocity of the
generated ions. When voltage is increased, more ions are formed at the corona electrode. At the same time, the attraction of these ions to the collector is increased due to the higher difference in potential. The force on the ions increases linearly with voltage. Likewise, the number of ions increases linearly with voltage. These two factors combine to produce an overall quadratic relationship between applied voltage and resultant thrust.

The main results from these experiments were referenced mainly to confirm our theoretical predictions. The direct comparison between the experimental values and simulation values will be discussed in Chapter 7.

5.4 Experimental Maximize Thrust Conclusions

Overall, the geometry of the EHD Lifters should be minimal in weight by using lightweight materials. The number of corners in the overall geometry depends on the size of the thruster and tradeoff regarding the weight of the structure and the chance of premature breakdown at the corners. The designs must be easy to construct and must maximize the usage of the weight towards thrust production.

For an optimal design, the frame of the thruster should be constructed in such a way that all of the supporting structure can be used to make thrust. In other words, the geometry of the design should allow for the entire frame to be covered in foil and to act as a thrust-producing EHD device. Solely using parts of the structure for strength or support decreases the overall efficiency of the thruster.

The collector surface should be smooth to ensure a uniform electric field and curved to the calculated radius of curvature to prevent thrust cancellation due to ionization at the collector. Compared to the other tested collector materials, aluminum foil has the best thrust-
to-weight ratio and is the least expensive for testing purposes. In addition, the aerofoil collector design produces more thrust than the cylindrical design.

Next, the minimum amount of aluminum foil should be used. Our experiments have shown that there is virtually no difference between using a sheet of aluminum foil that covers a large portion of the thruster and using a small piece of foil wrapped in a cylindrical shape. As long as the radius of the top of the foil is the same, there does not seem to be a difference in the thrust produced. Consequently, to minimize weight, the least amount of aluminum foil should be used.

Additionally, the corona wire should be as thin as possible and stretched as tightly as possible around the supporting structure. The small radius of the wire will create a large electric field at relatively small voltages allowing for a more efficient thruster. Stretching the wire tightly around its supporting structure ensures that the air gap between the corona wire and the collector will be uniform. Thus, maximum voltage can be applied without premature air breakdown due to localized areas of higher electric field.

Finally, one of the most important parts of optimizing an EHD Thruster design is precisely setting the air gap between the corona electrode and the collector. The air gap should be made such that the thruster is on the edge of electrical breakdown (i.e. making the air gap any smaller would cause breakdown). This maximized the electric field and thus the number of ions created for a particular voltage, which allows for maximum thrust. By applying the strategies proposed above, an efficient EHD Thruster can be designed with an optimal power to weight ratio.

Unfortunately, due to the strict safety requirements for performing high voltage experiments, the project was restricted to a limited amount of time and continued in
simulation form. Although educated trial-error improvements were conducted with promising results, simulations allow precise control of variables to extract exact properties based on initial modeling parameters.
Chapter 6:

EHD Governing Equations

In order to apply the EHD Thruster concepts as a simulation, partial differential equations (PDEs) must be used to represent the electrostatics and fluid dynamics (air flow) portions of the phenomenon. We will derive these equations and the boundary conditions under simplifying assumptions that should be applicable to all EHD Thruster models. This chapter will describe the equations necessary to represent EHD flow induced by corona discharge.

6.1 Electrostatic Equations

A simplified model of corona discharge is assumed in this study where the following apply:

- Only one species of ions is injected by the corona from the ionization layer to the drift layer
- The corona discharge is monopolar and stationary
- The ion mobility is constant and independent of the electric field’s influence
- Thermal diffusion of the ions is neglected
- At the boundary of the ionization layer, Kaptsov’s assumptions hold true
- Corona discharge is uniform across the corona’s surface
- The voltage at the corona and the surface of the ionization layer stays constant when CIV is reached
- The CIV is determined by Peek’s equation
- Manipulations of this equation will also determine the effective size of the ionization layer and voltage at this distance

Most literature [8,9,13,17] performing this calculation for ESP devices assumes that the ionization layer is small enough to be neglected. However, a corona wire with a small radius will have an ionization layer that is almost five times its size. This may still be a relatively small radius; however it is important to model this accurately in the simulation section.

When high voltage is applied to the EHD Thruster system, a space charge is formed, and a continuous electric current flows between the two electrodes. Both, applied voltage and space charge, will contribute to the generation of electric field between electrodes. The electric field intensity ($\vec{E}$) between the electrodes can be described by Gauss’s law

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho_{\text{ion}}}{\varepsilon_0}$$

(6.1)

where the equation relating electric field to potential is

$$\vec{E} = -\nabla \varphi$$

(6.2)

By substituting (6.1) with (6.2), we obtain Poisson’s equation, which is defined as

$$\vec{\nabla} \cdot \nabla \varphi = -\frac{\rho_{\text{ion}}}{\varepsilon_0}$$

(6.3)
where $\vec{E}$ is the electric field, $\rho_{ion}$ is the space charge density (C/m$^3$), \(\varphi\) is the scalar electric potential, and \(\varepsilon_0\) is the permittivity of free space (8.854x10^{-12} \text{ C/Vm}) [40]. This equation (6.3) relates the electric potential to the charge density and governs the electric field of the system.

The ionic charges are accelerated by the Coulomb force and move towards the collector. The charge drift creates an electric current with a density defined by the current density equation

$$j = \mu_p \rho_{ion} \vec{E} + \rho_{ion} \vec{U} - D \vec{\nabla} \rho_{ion}$$

(6.4)

where \(j\) is the ionic current density, \(\vec{U}\) is the velocity vector of airflow, \(D\) is the charge diffusion coefficients of ions (5.3x10^{-5} \text{ m}^2/\text{s}), and \(\mu_p\) is the ion mobility of positive ions (2.0x10^{-4} \text{ m}^2/\text{Vs}).

The drift layer has a combination of conduction (motion of ions under electric field relative to entire airflow), convection (transport of charges with airflow), and diffusion. Typically, the drift velocity of ions is usually two orders of magnitude larger than the velocity of the gas. In addition, the ion diffusion is of negligible importance compared to conduction [41]. With the conduction term dominant over convection and diffusion terms for the system, equation (6.4) simplifies to

$$j = \mu_p \rho_{ion} \vec{E}$$

(6.5)

Under steady state conditions, the current density must satisfy the charge conservation equation or current continuity equation
\[ \vec{\nabla} \cdot \vec{j} = 0 \]  
\hspace{2cm} (6.6)

After combining equations (6.5) and (6.6), substituting for the electric field with potential (6.2), expanding the divergence, and then substituting Poisson’s equation (6.3) with the result, the charge transport equation can be derived to be

\[ \vec{\nabla} \cdot (-\mu_p \rho_{ion} \vec{\nabla} \varphi) = \vec{\nabla} \rho_{ion} \cdot \vec{\nabla} \varphi + \rho_{ion} \vec{\nabla} \cdot \vec{\nabla} \varphi = 0 \]

\[ \therefore \vec{\nabla} \rho_{ion} \cdot \vec{\nabla} \varphi = \frac{\rho_{ion}^2}{\varepsilon_0} \]  
\hspace{2cm} (6.7)

This resulting equation is a dot product between the vector space charge density and scalar electric potential. The electric problem of corona discharge is governed by a set of two partial differential equations: Poisson’s equation (6.3) with unknown potential, \( \varphi \), and the charge transport equation (6.7) with the unknown space charge density, \( \rho_{ion} \).

Since the effective radius of the corona electrode is extended by the ionization layer, the model will only include the surface of the ionization layer with an applied voltage that follows the derived formula from Peek’s equation.

The space charge density in the ionization layer is negligible compared to that in the drift layer, therefore the external radius of the ionization layer can be approximated to increase proportionally to the inverse of the electric field strength: \( \frac{E_w}{E_o} \sim \frac{r_0}{r_w} \). Thus, Peek’s equation can be rewritten as

\[ r_0 = r_w \delta \left( 1 + \frac{0.0301}{\sqrt{r_w}} \right) \]
The external potential of the boundary of the ionization layer is dependent on the relationship between the radius and electric field, evaluated as

\[ V_0 = V_w - E_w r_w \ln \left( \frac{E_w}{E_0} \right) \]

Substituting a wire radius of 0.045 mm and an applied voltage of 20kV, the external radius of the ionization layer is calculated to be 0.22mm with a voltage of 18.8kV. Many papers [9,19,21] follow the assumption that the ionization layer is negligible, but notice that the ionization layer is almost five times larger than the original radius. Due to this size difference, the simulation model used the radius of the ionization layer instead of the actual wire radius.

Poisson’s equation (6.3) was solved within the drift layer, where the calculated \( V_0 \) is applied to the surface of the ionization layer. The boundaries of the computational section of air have their electric potentials set to zero. The potential on the ground collector is also set to zero. The charge transport equation (6.7) is also only applied to the drift layer. A zero diffusive flux is imposed on all boundaries except the surface of the ionization layer because the diffusion term does not affect the outflow boundaries.

Boundary conditions for space charge density are obtained by using Kaptsov’s assumption, which suggests that the electric field increases proportionally to the voltage below the corona onset level, but will preserve its value after the corona is initiated [34]. The initial value of the space charge density was adjusted such that the electric field strength was equal to the breakdown electric field strength in air, \( E_0 = 3.23 \times 10^6 \text{V/m} \).

Next, we will focus on the movement of particles, treating air as an incompressible fluid.
6.2 Fluid Dynamic Equations

To simplify the air flow model, the gas is assumed to be at room temperature and atmospheric pressure with a constant density and viscosity, a steady and laminar flow, and satisfies the continuity equation

\[ \nabla \cdot \vec{U} = 0 \]  

(6.8)

The conservation of momentum equation is also known as the Navier-Stokes equation, which is written in its full form as:

\[
\rho_{\text{air}} \left( \frac{\partial \vec{u}}{\partial t} + \vec{U} \cdot \nabla \vec{U} \right) = -\nabla p + \eta \nabla^2 \vec{U} + f_s \]  

(6.9)

where \( \rho_{\text{air}} \) is the air density (1.23 kg/m\(^3\)), \( p \) is the static air pressure (in Pascals), and \( \eta \) is the air dynamic viscosity (1.8205x10\(^{-5}\) Ns/m\(^2\)). The LHS of equation (6.9) represents the inertia per volume and the RHS represents the divergence of stress. Since the \( \frac{\partial \vec{u}}{\partial t} \) term is used for unsteady accelerations, only the convective acceleration term remains on the LHS.

The other body forces, \( f_s \), are represented as

\[
f_s = \rho_{\text{ion}} \vec{E} \frac{1}{2} \nabla^2 \vec{\varepsilon} + \frac{1}{2} \vec{\nabla} \left[ E^2 \left( \frac{\partial \varepsilon}{\partial \rho} \right) \rho \right] \]  

(6.10)

In this equation, the first term, \( \rho_{\text{ion}} \vec{E} \), is Coulomb force which is the force per unit volume on a medium containing free electrical charge. This is the strongest EHD force term, and, as with this case, dominates when DC electric fields are present. The second term, \( \frac{1}{2} E^2 \nabla \varepsilon \), is a dielectric force due to the force exerted on a non-homogeneous dielectric liquid by an electric field. This force is weaker than Coulomb force and usually dominates in an
AC electric field. The third term, \( \frac{1}{2} \nabla \left[ \mathbf{E}^2 \left( \frac{\partial \mathbf{E}}{\partial \mathbf{p}} \right)_T \right] \), is the electrostrictive term used for compressible fluids [41]. The gradient of a scalar is a modification to the fluid pressure, which acts in bulk between different liquids.

Since the second and third terms of the body force equation do not apply to the EHD Thruster formulation, the Coulomb force is the only body force affecting the system. After replacing the electric field with potential, the modified Navier-Stokes equation is as follows:

\[
\rho_{\text{air}} \mathbf{U} \cdot \nabla \mathbf{U} = -\nabla p + \eta \nabla^2 \mathbf{U} - \rho_{\text{ion}} \nabla \varphi \tag{6.11}
\]

The fluid dynamics boundary conditions are set such that no slip boundaries are used for all solid surfaces. Two electrodes act as stationary walls where the components of the velocity vectors vanish. The outside boundary is defined with a pressure inlet and outlet with both inlet and outlet pressures equal zero and outlet pressure in the direction of the back flow normal to the boundary. This means that the air is free to flow in both directions. The computation air space is set to an “open boundary,” which implies a zero normal stress (N/m\(^2\)) and pressure component.

Thus, the studied EHD flow model is described by the system of equations (6.3), (6.6), (6.7), (6.8), and (6.11) with the appropriate boundary conditions.
Chapter 7:

Thruster Modeling with COMSOL Multiphysics 3.4

The governing equations derived from the previous chapter were modeled with COMSOL Multiphysics 3.4. COMSOL Multiphysics is a modeling tool, simulator, and post processing software package that allows the cross-reference of multiple physics modules with the ability to represent any PDE into its system. Due to its many key features, this software was chosen to model the EHD Thruster system.

In this chapter, we will describe the modeling setup, equation simulation, and post-processing results of different EHD Thruster models. First, a simple 2-D thruster model was created for comparisons with the thrust production theoretical models and experiments discussed in earlier chapters. The reduced computational complexity of the 2-D model mainly reduced the time required to solve the system of equations and allowed for a wider range of variable adjustments. Manipulating the values for the radius of the corona wire, applied voltage, radius of collector, and air gap were automated and graphed in comparison to the results obtained from the experiments. The values that were optimized using the 2-D simulation were extruded to a 3-D single thruster. After testing a single thruster in 3-D, a full EHD lifter was constructed and tested.
Unfortunately, due to processor and memory limitations, the complexity of the 3-D EHD Thruster could not be solved for the velocity profile or electric field distributions. Creating a coarser mesh of the model by adjusting the element growth rate, mesh curvature factor, mesh cutoff, resolution of narrow regions, and maximum element size was tried to simplify the complexity of the thruster. However, the obtained answers from the coarser meshes had questionable accuracy and did not consistently follow the expected velocity patterns. By reducing the mesh density of the 3D model from ~8,000 elements to ~3,000 elements with local meshing parameters (mesh curvature factor, mesh cutoff, resolution of narrow regions for a selected region) near corners and curved surfaces, the program would finish the calculations of the PDEs without error, but there would be a noticeable decrease in the calculated thrust production. Since the corona discharge phenomenon is crucial to the produced thrust, finer meshes are required near the corona electrode in order to carefully represent the change in gradient for the Coulomb forces.

Graphical data representing the potential distribution, velocity profiles, and space charge densities were analyzed in all models. From these results, more specific design parameters were concluded.

7.1 COMSOL Software

The COMSOL Multiphysics software was specifically chosen for the GUI interface, flexibility, simple workflow, and large library of predefined physics equations. This provides an easy platform to solve complex, multiphysics simulation problems. Physics problems can be individually simulated on imported models with separate modules, which can then be combined to solve each of the variables cross-referenced between modules. This
means that the space charge density and electric field variables coupled between the electrostatic equations and fluid dynamics equations can be applied to an EHD Thruster model where the variables can be simultaneously solved.

7.1.1 COMSOL Modules

A module represents a particular PDE formula that will be solved for the drawn (or imported) model. Because the physics system is written with the generalized PDE coefficient form in equation:

\[
e^a \frac{\partial^2 u}{\partial t^2} + d \frac{\partial u}{\partial t} - \nabla \cdot (e \nabla u + au - \gamma) + \beta \cdot \nabla u + au = f
\]  

(7.1)

almost any physics equation can be modeled. The predefined list of physics equations is very extensive, and also provides examples used with each physics module. Some examples of applications designed, include the heat equation, conductive media, electrostatics, structural mechanics, acoustics, and fluid flow. The model can stack multiple modules with cross-referencing variables. These variables can either be solved simultaneously or used individually as initial conditions for other modules. For example, the calculated forces from Poisson’s equation were automatically updated as the body force variable in the Navier-Stokes equation.

7.1.2 COMSOL Workflow

In addition to the coupled variables feature, COMSOL also provides an intuitive workflow structure seen in Fig. 7.1. After drawing the model, modules can be added to
represent the governing equations. The model can then be discretized with the adjustable meshing methods, and then solved.

![Figure 7.1. Steps for COMSOL](image)

The post-processing component gives a large number of choices for graphical representations of all of the variables solved in the equations. These solved variables can be graphed directly onto the model and stacked to show multiple effects. The vector outputs particularly helped analyze the three-dimensional electric field and velocity profile relationships with the hard boundaries.

### 7.2 EHD Modules and Setup

The simulations in this project were run using COMSOL Multiphysics 3.4 software on a MacPro with Dual 2.8GHz Quad-core Xeon processors and 4GB of RAM. The electrostatics module, Navier-Stokes module, and a generalized PDE coefficient form module were used to represent the derived governing equations. Due to the generalized structure of COMSOL, the generalized PDE coefficient module could be used for the charge transport equation since a predefined module was not available. Although the governing
equations are quite similar, there are details in sub-domain equations and boundary conditions that require attention. Details of the COMSOL equations and modeling procedures are described in Appendix A.

7.3 2-D Single Thruster

A single 2-D EHD Thruster cross-section (Fig. 7.2) was modeled with the following parameters:

- \( R_{\text{wire}} = 0.045 \text{ mm} \)
- \( R_{\text{ionization}} = 0.22 \text{ mm (calculated)} \)
- \( R_{\text{collector}} = 0.75 \text{ mm} \)
- Air gap = 40 mm
- Foil length = 25 mm
- Air area = 0.52 m\(^2\)

The radius of the ionization layer was modeled instead of the radius of the wire following the simplifying assumptions made in the boundary conditions of the electrostatic equations (Section 6.1.3). A normal global mesh was chosen with the following parameters:

- Maximum element size = 0.2
- Element growth rate = 1.3
- Resolution of narrow regions = 1
- Mesh curvature factor = 0.3
- Mesh curvature cutoff = 0.005

This mesh contained approximately 1,200 elements. These thruster parameters were chosen to show the most prominent features in the post-processing plots.

Figure 7.2. 2-D single thruster model
The electrostatic module, PDE coefficient module for the charge transport equation, and Navier-Stokes module were applied separately to the 2-D thruster model. The variables in each of these modules were heavily dependent and needed to be solved separately before they were linked. Each of these sub-domains was set up in accordance to the details in Appendix A.

Figure 7.3. Electric potential for 2-D model

Fig. 7.3 and 7.4 show the potential and space charge density distributions in the air gap between the corona and collector electrodes. With a 20kV applied voltage to the corona and the collector at ground, a smooth potential distribution is found in the majority of the air gap except near the electrodes. Notice that the graph does not reach the maximum 20kV maximum voltage because corona discharge occurs before this point and maintains the CIV value along the external surface of the ionization layer. The space charge density has a peak within the air gap and decreases quickly towards the collector electrode. This will cause an electrical force that pulls the collector electrode upwards. The calculation for this value influenced by the electric field never exceeds the electric field intensity for air breakdown. For this configuration, the maximum space charge density was $7.72 \times 10^{-5} \text{ C/m}^3$. 
As seen in Fig. 7.5, the electric field is strong on the surface of both electrodes near the air gap. An electric field also forms at points of small radius of curvature, and causes gas ionization. The sharp closing connection in the aerofoil design creates the undesirable effect of a high electric field. This property could cause ionization of negative ions and cancel thrust production.

Fig. 7.6 shows the airflow velocity distribution as a result of the Navier-Stokes and charge transport modules. The airflow modeled in this simulation is mainly driven by the
Coulomb force. The transfer of momentum increases the airflow and peaks at the edges of the collector electrode.

Figure 7.6. Velocity distribution for 2-D model

The Coulomb force distribution in Fig. 7.7 is a product of the electric field and space charge density. The Coulomb force drives the air motion and is seen to be greatest between the corona and collector.

Figure 7.7. Coulomb force

The pressure distribution (Fig. 7.8) is consistent with the velocity distribution. The part of the wire that faces the collector electrode experiences a maximum negative relative pressure due to the highest acceleration of gas molecules. The maximum positive pressure
occurs at the rounded surface where the airflow decreases around the aerofoil shape. This area will cause the negative thrust production from the airflow.

The total force in this system is a direct result of the airflow caused by the Coulomb force subtracted by the sum of forces from the viscous drag and pressure. The main force contributing to the levitation is the longitudinal component (along the same direction as a straight line between the wire and collector). To calculate the value of the Coulomb force, the Coulomb longitudinal component distribution was integrated over the whole computational domain.

COMSOL Multiphysics provides an integration tool, which solved: Space Charge Density (C/m$^3$)$\times$Electric Field (V/m or N/C) = Coulomb Force (N/ m$^3$). For this particular configuration, the total force was calculated to be 0.13N/m. The pressure force pushes down on the thruster, which reduces the overall levitation force. For this configuration, the sum of pressures was solved to have a negligible contribution of approximately $10^{-7}$ N/m, which leads to a dominant Coulomb force. At 20kV, approximately 11g can be lifted with the air
velocity generated by the EHD phenomenon. This thrust production should include the EHD lifter’s own weight with some additional payload.

7.4 2-D Single Thruster comparison

After confirming that the force from the simulation produces a reasonable amount of lift, the thruster parameters were altered to match the thruster experiments from Chapter 5. Fig. 7.9 shows a 2-D thruster with a collector radius of 3.5mm and an air gap of 25mm.

![Figure 7.9. Single Thruster for comparison](image)

The voltage applied to this system was varied from 8kV to 24kV in steps of 2kV. A MATLAB program was written to automate the process of adjusting voltages and outputting integrated calculations of the Coulomb force. The only issue with this program was the inability to halt when the breakdown voltage of air was reached by the space charge density without completely relying on Peek’s formula. This situation was avoided by using the applied voltage range from the experimental results in Chapter 5. Since environmental factors, such as humidity, should decrease the breakdown value of air, the voltages measured should be below the air breakdown threshold. As seen in the Force vs Voltage graph in Fig.
7.10, the simulated results follow the quadratic curve as voltage increases, in accordance to the theoretical approximations made in Barsoukov’s model from Chapter 4.

![Force vs Voltage](image)

Figure 7.10. Force vs Voltage comparison

It is important to note that in Fig. 7.10, the force output is much larger than those measured in the experiment. This could have been an effect from the pressure forces, but is more likely explained by the human error while conducting the experimental cases. When the corona wire is adjusted with different air gaps, the wire must be taut and should be completely horizontal to the collector surface. Although the single thruster framework setup has avoided a “z-offset” and allows for varied air gaps, it is impossible to be exact. Environmental factors, such as humidity and dusty particles present, can lower the electric field strength of air and cause air breakdown well below the approximations proposed in Peek’s equation.

The scale measuring the force produced by the thrusters was covered with aluminum foil to create an effective Faraday cage that would prevent the influence of strong electric fields from the applied electric potential. Even though this may have seemed effective with
reducing the measurement variations, the scale could have malfunctioned from over-
exposure or left-over charge interferences. After conducting these experiments, it was
hypothesized that there would have been a voltage leakage through the balsa wood structures.
For future experiments, the corona wire should be separated from the structure with an
insulation material to prevent this voltage leakage phenomenon.

![Figure 7.11. Comparison electric field and space charge density](image)

Comparing the surface plots of the electric field and space charge density between the
experimental setup (Fig. 7.11) and the first experiment (Fig. 7.3 and 7.4), there is a difference
between the intensities present on the collector’s surface. A small collector radius is very
difficult to accurately build for experimental reasons, thus the air gap must be smaller to
increase the thrust produced by the potential difference. Unfortunately, before a larger thrust
can be produced, an electric field forms on the surface of the collector facing the wire and
causes a very small threshold for electrical conductivity of air. Due to the decreased window
of operation for the voltage, a smaller velocity field is present as seen in Fig. 7.12. The force
generated by this model was only 0.06N/m (after dividing through the length of the tested
thruster), which was enough to lift approximately 5.2g.

74
The radius for the collector and corona wire were mainly chosen for comparison purposes; however these values were also optimized in terms of the limitations of the materials available. The corona radius was chosen to match the smallest wire radius in the lab, and the collector radius was the smallest radius that can be maintained by the aluminum foil. In most application cases, the size of the collector and wire would be fixed, while the only possible flexible parameter is the air gap. Therefore, to test the optimization of the EHD Thruster, it is justified that only the air gap was varied while the other variables act as a control. Optimizing the dimensionality of the collector radius size, foil length, z-offset, or many other factors would exponentially increase the state space of testing.

The resulting Force vs Voltage graphs (Fig. 7.13 and 7.14) show an expected trend confirming theoretical assumptions. The larger air gap yields a lower force at lower applied voltages, but maintains a larger window of operability to obtain an overall higher maximum thrust production. This is also supported by Fig. 7.15, which shows the force output given the breakdown voltages at specific air gaps.
The larger air gap option is not energy efficient; however, the main focus is to obtain a reasonable amount of thrust for propulsion applications. There exists a balance where the maximum thrust can be reached within a reasonable voltage. Simulations, where all of the other variables are kept controlled while one is altered, seem to be the best way of optimizing the thruster parameters. This does not necessarily express parameters that may be interlinked, but it offers a more reasonable state space for analysis.

Figure 7.13. Force vs Voltage for air gaps

Figure 7.14. Force vs Voltage for air gaps
Additional tests with the air gap have followed with the fixed radius for the collector, radius for the corona, foil length, materials, and aerofoil collector shape. The maximum force produced by this configuration within a practical power supply limitation was 41.64mN with a 5.0cm air gap at 34kV. The graph on the right of Fig. 7.15 shows the force vs air gap decaying relationship at a fixed voltage of 16kV. Although a polynomial curve fits this graph, theoretical models show this to represent a ln(x)/x curve.

<table>
<thead>
<tr>
<th>Air gap (cm)</th>
<th>Breakdown Voltage (kV)</th>
<th>Force (mN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0</td>
<td>16</td>
<td>20.25</td>
</tr>
<tr>
<td>2.5</td>
<td>22</td>
<td>33.72</td>
</tr>
<tr>
<td>3.0</td>
<td>24</td>
<td>33.15</td>
</tr>
<tr>
<td>3.5</td>
<td>26</td>
<td>33.21</td>
</tr>
<tr>
<td>4.0</td>
<td>30</td>
<td>39.99</td>
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<tr>
<td>4.5</td>
<td>32</td>
<td>40.80</td>
</tr>
<tr>
<td>5.0</td>
<td>34</td>
<td>41.64</td>
</tr>
</tbody>
</table>

Figure 7.15. Force vs Air gap

Optimizing the air gap variable is difficult when only given the restriction of finding the maximum thrust because a larger air gap will yield a larger maximum thrust capability. However, given voltage limitations and threshold operational values, a relationship between the air gap and the voltage production can be formed. For example, if the thruster must maintain a large operational flexibility below a specific voltage, the optimal air gap can be extrapolated by weighing the rewards of the restrictions. In this case, the optimal air gap of 3.5 cm was concluded due to the breakdown voltage restriction and gradual ascent in the force vs voltage curve.
Due to time restrictions, the air gap was the only variable extensively tested with the simulator. Other variables will be explored with future simulation models. More importantly, a 3-D model focusing on the higher dimensionality of the differential equations could yield more unique results.

7.5 3-D Single Thruster

Using the optimal 3.5cm air gap chosen from the previous 2-D thruster configuration and a 3.5mm collector radius, the 3-D thruster model was extruded and the physics equations were applied. The initial aerofoil collector shape from the 2-D model was tested, but yielded computationally intensive complexities at the sharp edges during the solving step. Therefore, to simplify this system, the cylindrical-shaped collector seen in Fig. 7.16 was applied. The 3-D model maintains the same type of modules, sub-domain settings, and boundary conditions that existed in the 2-D model. The transformation to the three-dimensional problem did not pose any difficulty for applying the physics, but the meshing and solving steps were more time consuming.

Figure 7.16. 3-D Single thruster
After solving the system of equations, the electric potential (Fig. 7.16), space charge density (Fig. 7.17), and electric field (Fig. 7.18) were graphed. Although a 3-D graphing system directly on the computational domain was available through the program, it was difficult to represent the details. The slices used for these representations showed the cross-section of the main axes for the collector and a vertical slice for the profile view.
Figure 7.18. 3-D Single thruster space charge density

The electric field representation in Figure 7.19 was especially difficult to conclude any particular advantages or disadvantages without more complex collector curves. The visualization of the forces interacting between the electrode surfaces was interesting, but quantitatively uninformative.

Figure 7.19. 3-D Single thruster electric field

More complex collector shapes were tested, but the high density meshes at the sharp points began pushing the limit of the computer’s resources. Since the velocity distribution
requires the most calculations linking the space charge density and electric field, the simulation did not converge for the more complex domain. The meshing tool was explored to use coarser separations, but this degraded the quality of the results and returned plots that did not resemble the expected curves.

The data used to calculate the Coulomb force approximation does not directly utilize the velocity distribution since the pressure and viscous forces are negligible. The total thrust generated by the 3-D single thruster extruded to 0.3m was calculated to be 0.02N, which can lift approximately 1.9g. Considering the successful proof-of-concept flight weighed approximately 1.3g and measured 0.3m extended, the simulation maintains a reasonable approximation.

### 7.6 3-D Triangle Lifter

Although the computational complexity of the 3-D triangle lifter would have difficulty computing the 3-D Navier-Stokes module, this model (shown in Fig. 7.20) was created to hopefully maintain an approximation for the thrust production and electric field distributions. Although the electrical potential distribution in Fig. 7.21 was attainable, the 3-D PDE module for the space charge density led to the error, “Out of memory during LU factorization.” A coarser mesh with less emphasis around the sharp corners was tested, but the computer was still not powerful enough to perform the task.
Overall, the simulation results matched the expected properties derived from theoretical designs and tested in the laboratory. Both of the Force vs Voltage curves calculated from the simulations and measured in experimentation followed a quadratic
increase due to the combination of increased ion production and increased Coulomb force acting on the ions. Due to the ideal environmental conditions and EHD Thruster setup in the COMSOL Multiphysics simulations, the results for the simulator showed a higher thrust production than those from the experiment.

The simulation results did not successfully map the optimized EHD Thruster design due to the complex dimensional interactions between the thruster variables. The air gap tests conducted for a particular configuration of a practical EHD Thruster were only optimized within stricter constraints and reward goals. Unfortunately, limitations in software restricted the desired testing with nesting and stacking properties. Complex electric field and velocity distributions in these 3-D models would have been useful to visualize and could have shown interesting properties near the sharp corners. Given more memory and a faster processor, the model simulation built in this paper could be extended to calculate the thrust production for a given model.
Chapter 8:

Overall Conclusions

Starting from a basic fundamental physics approach, a series of theoretical calculations, physical experimentation, and simulations were conducted in this paper to more closely understand the EHD Thruster variables involved with optimizing the thrust production from the EHD phenomenon. Significant research and testing have led to a list of optimization parameters and useful tips that would increase thrust production. The basic simulation framework provides for a large variation of tests that could be extended to more closely analyze the effects of each parameter. More powerful computers may be able to calculate the thrust production and show the electrostatic and hydrodynamic distributions for a given 3-D thruster model.

8.1 Optimization Parameters

After understanding the fundamental properties of the EHD phenomenon and an applicable theoretical model, a set of design conclusions can be extended for experimental tests. When building an EHD Thruster, it is important to reduce the structural weight by utilizing the full surface for thrust production. Materials such as balsa wood and aluminum foil, are not only less expensive and more readily available, but tested to be the most
effective. The corona wire radius should be as small as possible while maintaining durability to reduce the voltage at which corona discharge occurs. The structural material should be light-weight and act as an insulator to prevent voltage leakage. Rounded surface edges for the top and bottom parts of the collector are required to prevent a reverse thrust cancellation from ionization at the collector. Reducing the corners in the thruster geometry will reduce the localized areas of higher electric field, but for experimental purposes, the more important focus should be the accurate construction of the model. A taut corona wire parallel and directly above the collector will remove the effects of a “z-offset” and premature air breakdown.

Based on each specific configuration of the EHD Thruster, a different solution will exist for the air gap optimization. It is the high dimensionality of this problem that makes testing very difficult and time consuming. The solution for this continuous problem must be discretized, compared, and then fine-tuned to search for the optimal design configuration for each combination of variables. Exploration of the full state space of variables would be impossible, but keeping some dimensions fixed may form a smaller range of combinations. Once the simulation of this EHD Thruster problem is completely defined, a more mathematically optimal solution can be solved.

The 3-D model simulations were setup accurately, but limited by our hardware. The algorithms required to run the calculations obtained the expected results with a small variation. A coarser mesh could have been applied to the models for a completed 3-D view of the system, but the corona discharge causing the ion-wind generation occurs in great detail near the surface of the thin corona electrode. A finer mesh must exist near these areas or the force calculations from the velocity plots would decrease by 0.002N/m.
Barsoukov’s theoretical model, the measured experimental results, and the electrical and mechanical governing equations that defined the simulations all produced consistent force calculations. The single thruster experiment \((V=2.5\times10^4V, L=0.2m, d=0.025m, r_w=4.5\times10^{-5}m, r_c=1\times10^{-3}m)\) from senior projects was measured to produce 0.0062N. Applying Barsoukov’s model yielded a generated force of 0.0076N, and modeling the thruster with COMSOL yielded a total force of 0.011N. Although these values aren’t perfect, staying within this standard deviation is very good considering the sources of error and different methods of calculating the effective force. Experimental inaccuracies due to high electric field interference of the scale or human error in the construction of the thruster could have caused major discrepancies in the result and can yield the decrease of measured force by a factor of 2 from the simulations. The simulation was expected to yield the largest force since one of the frictional forces was ignored in the calculation and the conditions were exactly set to a measured, average breakdown voltage. The PDE governing equations provide a higher level of accuracy within the simulation than measured in the experiments conducted in senior year.

Applying Barsoukov’s model to the single thruster framework yielded force calculations that were +/- 0.005N with respect to the simulation results. This works considerably well for the single thruster frame, but Barsoukov’s model does not account for the complexities in the corners of the EHD lifter devices. The experimental proof-of-concept triangle lifter built and tested during senior year produced a force of approximately 0.0021N at 13kV. If the length substituted into Barsoukov’s model is the sum of the length of the sides’ thrusters \((L=0.17m)\), then the force produced with the same power variables \((V=13kV, d=0.02m, r_w=4.5\times10^{-5}m, r_c=1\times10^{-3}m)\) is 0.0029N. This over-estimate could have been a
result of thrust interference at the corners. The shared bulk air by the adjacent sides near the corners could have reduced the overall thrust production.

The simulation force calculations confirmed the experimental measurements, but more importantly proved that, for a given wire radius, a balance between the air gap and the radius of collector exist to obtain the optimal thrust before air breakdown. However, this optimization problem converges only when more restrictions are applied to the system, such as a maximum applied voltage, maximum current, and level of control. It is recommended that these parameters are retested for each particular EHD application. Goals, such as improving power efficiency or maintaining a steady control of thrust, will require different parameter settings.

### 8.2 Future Work with Simulation

Due to time constraints and limitations in hardware, the more advanced variable tests could not be properly performed. The upcoming COMSOL Multiphysics 3.5 version optimizes multiple-core processor calculations, and may be a viable solution to improve speeds for future experiments.

There is still a great possibility of useful results for areas such as, studying the effects of the environment, determining the scalability of EHD Thrusters, and testing more complex geometries. The simulation provides humidity and pressure settings within the computational domain, which may affect the operation of the thrusters. The complex geometries can be studied to calculate the electric field interactions and velocity distributions, which may be useful for designing EHD Thrusters that utilize stacking or nesting.
The COMSOL Multiphysics software also provides a COMSOL Script tool. Similar to MATLAB m-files, the scripting tool is a platform that can increase the efficiency of conducting and calculating simulation results. In the most basic sense, a sub-routine can be written to vary the air gap and radius of collector sizes to calculate the overall thrust production, in order to find correlations between the design parameters for the specific thruster. In addition, opening this software to programming practices allows the use of exploration methods and algorithms (e.g., dynamic programming) to search the state space represented by the EHD Thruster variables. Once the continuous state space (consisting of the air gap, radius of collector, foil length, and radius of wire) is separated into a discrete dimensionality, an optimal combination of these variables exists to obtain any optimized parameter (e.g., thrust or energy efficiency).

Ideally, an interface for the existing simulation tool would be extended to make this process more efficient. Lifter simulations [29] are available where the parameters of the thruster in its most fundamental form can be inserted into a set of equations to solve for the thrust production. However, these theoretical results do not involve the derived governing equations, which accounts for the specific geometries provided in CAD or SolidWorks models. In this interface, the user can upload or draw a specific thruster model, assign the positive and negative applied voltages directly to the model, and ultimately output the thrust produced by the system. The velocity distribution and electric field interactions in this simulation would be presented for analysis. This would allow for thorough testing of different geometries and a detailed study of the airflow distributions, which may lead to new potential thruster designs.
The materials library included in the simulation software is very extensive and should be tested for the most efficient material that yields the highest thrust-to-weight ratio. Manually finding these materials and building accurate representations of these thrusters would be unrealistic. Many of the complex shapes and designs can also be tested without extensive exposure to the safety hazards (e.g., ozone production and high voltage risks) that accompany tests with corona discharge.

### 8.3 Closing Statements

The thrust produced by the corona discharge is essentially the airflow mainly influenced by the Coulomb’s force. The force production calculated in the experiments in this paper showed a theoretical, practical, and simulated ability to lift the weight of a typical EHD lifter. Unfortunately, the “Back to the Future” Hoverboard dream will not be fulfilled by this method due to the low thrust production and safety concerns. Optimizations in the EHD Thruster design to improve this thrust production were unable to significantly increase the results. The strong electric field interferences and ozone production as a direct result of the corona discharge method would be difficult to pass the standard safety regulations. Although propulsion theory may not be completely feasible for a means of travel, the EHD concepts and simulations studied in this thesis can be slightly altered for applications in ESP, EHD speakers, and EHD pumping devices.
Appendices
Appendix A:

COMSOL Multiphysics 3.4 EHD Equations

The EHD Thrusters were modeled in COMSOL Multiphysics using the governing equations derived in Chapter 6. To assist with future replications of this model, this appendix lists the detailed implementation of these equations. Detailed steps in the workflow and possible alternative solutions are discussed.

A.1 Module Equation Formulations

Electrostatics Module Formulation:

\[ -\nabla \cdot d\varepsilon_0 \varepsilon_r \nabla V = d\rho \]

To match this to Poisson’s equation:

\[ \nabla^2 \varphi = -\frac{\rho_{ion}}{\varepsilon_0} \]

Sub-domain Settings:

- Thickness, \( d = 1 \) m
- Space charge density, \( \rho = u \) (from PDE module) C/m\(^3\)
- Relative permittivity, \( \varepsilon_r = 1.00054 \)
- Permittivity of vacuum, \( \varepsilon_0 = 8.85418 \times 10^{-12} \) F/m
Boundary Conditions:

- External boundary of ionization layer, \( V \sim 20\text{kV} \) (depends on test conditions)
- Boundary of collector, \( V = 0 \)
- Not applied to the boundaries of the analysis area

PDE, Poisson’s Equation Module:

Assuming the diffusion is negligible, the charge transport equation only relies on the electric potential.

\[
\nabla \cdot (-c \nabla u) = f
\]

To match this to the charge transport equation:

\[
\nabla \rho_{ion} \cdot \nabla \phi + \rho_{ion} \nabla \cdot \nabla \phi = 0
\]

\[
\nabla \rho_{ion} \cdot \nabla \phi = \frac{\rho_{ion}^2}{\varepsilon_0}
\]

Sub-domain Settings:

- Diffusion coefficient, \( c = V \) (from Electrostatics module)
- Source term, \( f = \frac{u^2}{\varepsilon_0} \) (predefined in Electrostatics module)

Boundary Conditions:

- Initial conditions for the space charge were set through trial and error methods where the maximum electric field intensity was less than the air breakdown intensity.
Incompressible Navier-Stokes Module:

\[ \vec{\nabla} \cdot \vec{\U} = 0 \]

\[ \rho \vec{\U} \cdot \vec{\nabla} \vec{\U} = \vec{\nabla} \cdot \left[ -pI + \eta \left( \vec{\nabla} \vec{\U} + (\vec{\nabla} \vec{\U})^T \right) \right] + F \]

To match this to our Navier-Stokes Equation:

\[ \rho_{air} \vec{U} \cdot \vec{\nabla} \vec{U} = -\nabla p + \eta \vec{\nabla}^2 \vec{U} - \rho_{ion} \nabla \varphi \]

Sub-domain Settings:

- Density, \( \rho = 1.23 \text{ kg/m}^3 \) (of air)
- Dynamic viscosity, \( \eta = 1.8 \times 10^{-5} \text{ Pa*s} \)
- Volume force in x-dir, \( F_x = \rho_{emes} \vec{E}_x \text{ N/m}^3 \) (body force from Coulomb’s force)
- Volume force in y-dir, \( F_y = \rho_{emes} \vec{E}_y \text{ N/m}^3 \)

Boundary Conditions:

- Open boundary set for boundaries of the analysis space
- Not applied to corona or collector

A.2 Modeling Steps

The general modeling steps for the COMSOL Multiphysics software are as follows:

1. Select a module (predefined in the library)
2. Draw the model
   a. Use the draw mode (you can merge and subtract objects)
   b. Import models from CAD or SolidWorks
3. Enter the physics of the module
   a. Physics >> Subdomain Settings
i. The subdomains are numbered and can be named by Groups (for the EHD models, Air, Foil, and Wire labels are appropriately grouped in all of the modules). You can select directly from the model.

ii. The subdomain is the area enclosed by the geometry. Specific physics settings can be set for each group

b. Physics >> Boundary Conditions

i. Boundaries are the individual lines drawn. Once again, the grouping function applies

ii. Set the initial conditions for the boundary (e.g., set electric potential of surface of ionization layer)

c. Calculating space charge density

i. Trial and error alteration of the initial condition constant such that the electric field did not exceed the air breakdown value of approximately $3.23 \times 10^6$ V/m. If this value could not be reached, then there would be an electrical breakdown

4. Make a mesh

   a. Normal meshing is defaulted, but advanced properties for refining meshing parameters are possible.

   b. Finer meshing is better because the accuracy is important for the electric field but this requires more time and resources to solve

5. Solve the module

   a. Solve>>Solver Parameters

   b. Modules can be solved in any group or sequence

   c. Each of the variables can be solved separately for troubleshooting

6. Post-processing

   a. Postprocessing >> Plot Parameters

   b. Set the plot type

      i. The most common is Surface to see a distribution of the results directly on the model

7. Calculating forces
a. Integration tool
   i. Coulomb force (N/m$^3$) = Space charge density (C/m$^3$) * Electric field (V/m or N/C) over the full computational domain of the air
   ii. Longitudinal component of the Pressure
Bibliography


pp. 48-87.


